IISER BERHAMPUR

BS-MS SYLLABUS

Mathematical Sciences

I Semester

Course No.	Course Name	LecHr	Lab Hr	Tut Hr	SS Hr	Credit
BIO 101	Introduction to Biological Sciences	3	0	1	6	3
BIO 103	General Biology Laboratory I	0	3	0	0	1
CHM 101	General Chemistry	3	0	1	6	3
CHM 103	General Chemistry Laboratory	0	3	0	0	1
CDS 101	Introduction to Computers	2	1	0	6	3
HSS 103	Basics of Communication Skills	1	0	0	2	1
MTH 101	Introduction to Mathematics	3	0	1	6	3
PHY 101	Mechanics	3	0	1	6	3
PHY 103	Mathematical Methods	1	0	1	0	1
EES 101	Introduction to Earth System Sciences	3	0	1	6	3
Total		19	07	06	38	22

II Semester

Course No.	Course Name	LecHr	Lab Hr	Tut Hr	SS Hr	Credit
BIO 102	Biochemical and Cellular basis of life	3	0	1	6	3
BIO 104	General Biology Laboratory II	0	3	0	0	1
CHM 102	Basic Inorganic Chemistry	3	0	1	6	3
CHM 104	Inorganic Chemistry Laboratory I	0	3	0	0	1
HSS 104	Oral and Written Communication	1	0	0	2	1
EES 102	Introduction to Environmental Sciences	3	0	1	6	3
MTH 102	Calculus of One Variable	3	0	1	6	3
PHY 102	Electromagnetism	3	0	1	6	3
PHY 104	General Physics Laboratory I	0	3	0	0	1
Total		16	09	05	32	19

III Semester

Course No.	Course Name	LecHr	Lab Hr	Tut Hr	SS Hr	Credit
BIO 201	Introduction to Genetics and Evolution	3	0	1	6	3
BIO 203	General Biology Laboratory III	0	3	0	0	1
CHM 211	Basic Organic Chemistry	3	0	1	6	3
CHM 213	Organic Chemistry Laboratory I	0	3	0	0	1
EES 201	Foundation of Earth Sciences: Part 1 (Introduction to Mineralogy, Petrology)	3	0	1	6	3
HSS 209	Technical Writing	2	0	0	4	2
MTH 201	Linear Algebra	3	0	1	6	3
PHY 201	Waves and Introductory Optics	3	0	1	6	3
PHY 203	General Physics Laboratory II	0	3	0	0	1
Total		17	09	05	34	20

IV Semester

Course No.	Course Name	LecHr	Lab Hr	Tut Hr	SS Hr	Credit
BIO 202	Molecular Biology and Developmental Biology	3	0	1	6	3
BIO 204	General Biology Laboratory IV	0	3	0	0	1
CHM 222	Classical Thermodynamics	3	0	1	6	3
CHM 224	Physical Chemistry Laboratory I	0	3	0	0	1
EES 202	Foundation of Earth Sciences: Part 2 (Introduction to Rock Deformation and Plate Tectonics)	3	0	1	6	3
HSS 207	Macroeconomics	1	0	0	2	1
MTH 202	Multivariable Calculus	3	0	1	6	3
PHY 202	Quantum Physics	3	0	1	6	3
PHY 204	General PHY Lab III	0	3	0	0	1
CDS 202	Information for Science and Technology	2	0	0	4	2
Total		18	09	05	36	21

V Semester

Course No.	Course Name	LecHr	Lab Hr	Tut Hr	SS Hr	Credit
MTH 301	Group Theory	3	0	0	9	4
MTH 303	Real Analysis I	3	0	0	9	4
MTH 305	Elementary Number Theory	3	0	0	9	4
MTH ***	Departmental Elective I	3	0	0	9	4
*** ***	Open Elective I	3				3/4
Total		**15**				**19/20**

VI Semester

Course No.	Course Name	LecHr	Lab Hr	Tut Hr	SS Hr	Credit
MTH 302	Rings and Modules	3	0	0	9	4
MTH 304	Metric Spaces and Topology	3	0	0	9	4
MTH 306	Ordinary Differential Equations	3	0	0	9	4
MTH ***	Departmental Elective II	3	0	0	9	4
*** ***	Open Elective II	3				3/4
Total		**15**				**19/20**

VII Semester

Course No.	Course Name	LecHr	Lab Hr	Tut Hr	SS Hr	Credit
MTH 401	Fields and Galois Theory	3	0	0	9	4
MTH 403	Real Analysis II	3	0	0	9	4
MTH 405	Partial Differential Equations	3	0	0	9	4
MTH 407	Complex Analysis I	3	0	0	9	4
*** ***	Open Elective III	3				3/4
Total		**15**				**19/20**

VIII Semester

Course No.	Course Name	LecHr	Lab Hr	Tut Hr	SS Hr	Credit
MTH 404	Measure and Integration	3	0	0	9	4
MTH 406	Differential Geometry of Curves and Surfaces	3	0	0	9	4
MTH ***	Departmental Elective III	3	0	0	9	4
*** ***	Open Elective IV	3				3/4
*** ***	Open Elective V	3				3/4
Total		**15**				**18/20**

IX Semester

Course No.	Course Name	LecHr	Lab Hr	Tut Hr	SS Hr	Credit
MTH 501	Project Work					16
MTH 503	Functional Analysis	3	0	0	9	4
MTH 599	Reading course					4
HSS 503	Law Relating to Intellectual Property and Patents	1	0	0	2	1
Total		**10**				**25**

X Semester

Course No.	Course Name	LecHr	Lab Hr	Tut Hr	SS Hr	Credit
MTH 501	Project Work					16
MTH 599	Reading course					4
HSS 504	Law Relating to Intellectual Property and Patents	1	0	0	2	1
Total		**10**				**21**

BS-MS SYLLABUS, MATHEMATICAL SCIENCE

MTH 101: Introduction to Mathematics (3)

***Course Contents*:**

Method of Mathematical Proofs: Induction, Construction, Contradiction, Contrapositive

Set: Union and Intersection of sets, Distributive laws, De Morgan's Law, Finite and infinite sets.

Relation: Equivalence relation and equivalence classes.

Function: Injections, Surjections, Bijections, Composition of functions, Inverse function, Graph of a function.

Countable and uncountable sets, Natural numbers via Peano arithmetic, Integers, Rational numbers, Real Numbers and Complex Numbers. Sequences and series of real and complex numbers.

Matrices, Determinant, Solving system of linear equations.

Symmetry of Plane Figures: Translations, Rotations, Reflections, Glide-reflections, Rigid motion.

*If time permits: Divisibility of integers.

***Suggested Books*:**

- G. Polya, "How to Solve It", Princeton University Press, 2004.
- K. B. Sinha et. al., "Understanding Mathematics", Universities Press (India), 2003.
- M. Artin, "Algebra", Prentice-Hall of India, 2007 (Chapters 1, 4, 5).
- J. R. Munkres, "Topology", Prentice-Hall of India, 2013 (Chapter 1).
- R. Goldberg, "Methods of Real Analysis", Oxford & IBH Publishing Co. Pvt. Ltd, New Delhi, 1976.
- R. G. Bartle and D. R. Sherbert, "Introduction to Real Analysis", John Wiley & Sons, 1992.

MTH 102: Calculus of One Variable (3)

***Learning Objectives*:**

This is a core mathematics course for first-semester BS-MS students. The course introduces the basic concepts of differential and integral calculus of one real variable with an emphasis on careful reasoning and understanding of the material.

***Course Contents*:**

Introduction to the real number system, field axioms, order axioms and the completeness axiom

Sequences and series of numbers, convergence of a sequence, Cauchy's criterion, limit of a sequence, supremum and infimum, absolute and conditional convergence of an infinite series, tests of convergence, examples

Limits and continuity, definitions, continuity and discontinuity of a function at a point, left and right continuity, examples of continuous and discontinuous functions, intermediate value theorem, boundedness of a continuous function on a closed interval, uniform continuity

Differentiation, definition and basic properties, Rolle's theorem, mean value theorem, Leibnitz's theorem on successive differentiation, Taylor's theorem

Integration, Riemann integral viewed as an area, partitions, upper and lower integrals, existence of the Riemann integral, basic properties, fundamental theorem of integral calculus, integration by parts, applications

***Suggested Books*:**

- G. B. Thomas and R. L. Finney, *Calculus and Analytic Geometry*, 9th edition, Indian student edition, Addison-Wesley, 1998
- T. M. Apostol, *Calculus,* Volumes 1 and 2, 2nd edition, Wiley Eastern, 1980
- R. Courant, F. John, *Introduction to Calculus and Analysis,* Volume 1, Classics in Mathematics, Springer, 1989

MTH 201: Linear Algebra (3)

***Learning Objectives*:**

This is the second core course in calculus designed for second year BS-MS students. The course deals with the multivariable calculus of vectors in dimension 2 and higher. The course concludes with an introduction to first order ODEs, and their solutions.

***Course Contents*:**

Review of complex numbers

Matrices, matrix operations, special matrices (diagonal, triangular, symmetric, skew-symmetric, orthogonal, hermitian, skew hermitian, unitary, normal), vectors in $\mathbf{R}^n$ and $\mathbf{C}^n$, matrix equation $\mathbf{Ax = b}$, row-reduced echelon form, row space, column space, and rank of a matrix. Determinants. Systems of linear equations

Vector space $\mathbf{R}^n$, linear independence and dependence, linear span, linear subspaces, bases and dimensions

Vector spaces, bases and dimensions, linear transformations, matrix of a linear transformation, rank-nullity theorem

Inner product spaces, orthonormal bases, Gram-Schmidt orthogonalization, projections

Eigenvalues and eigenvectors of a linear operator, characteristic polynomial, diagonalizability of a linear operator, eigenvalues of the special matrices stated above, spectral theorem for real symmetric matrices and its application to quadratic forms, positive definite matrices

***Suggested Books*:**

- T. M. Apostol, *Calculus,* Volume 2, 2nd edition, Wiley Eastern, 1980
- H. Anton, *Elementary linear algebra and applications,* 8th edition, John Wiley, 1995
- G. Strang, *Linear algebra and its applications,* 4th edition, Thomson, 2006
- S. Kumaresan, *Linear algebra - A Geometric Approach*, Prentice Hall of India, 2000
- R. Rao and P. Bhimasankaram, *Linear Algebra*, 2nd edition, Hindustan Book Agency, 2000
- M. Artin, *Algebra*, Prentice-Hall of India, 1994
- R. Bapat, *Linear Algebra and Linear Models,* HBA, 1999

MTH 202: Multivariable Calculus and Differential Equations (3)

Vectors in $\mathbf{R}^3$, dot product of vectors, length of a vector, orthogonality of vectors, cross product of vectors

Lines, planes, and quadric surfaces

Continuity and differentiability of vector-valued functions, tangent vectors

Functions of two or more variables, limits and continuity, partial derivatives, gradient, directional derivatives, maxima, minima and saddle points, Lagrange multipliers

Double and triple integrals, change of coordinates, vector fields, line integrals, surface integrals, Green's theorem, Divergence theorem, Stokes' theorem

First order ordinary differential equations: variables separable, homogeneous, linear and exact equations

***Suggested Books*:**

- G. B. Thomas and R. L. Finney, *Calculus and Analytic Geometry,* 9th edition, Indian student edition, Addison-Wesley, 1998
- T. M. Apostol, *Calculus,* Volumes 1 and 2, 2nd edition, Wiley Eastern, 1980
- J. E. Marsden and A. Tromba, *Vector Calculus,* W.H. Freeman & Company, 2004
- R. Courant, F. John, *Introduction to Calculus and Analysis,* Vol. 2, Classics in Mathematics, Springer, 1989

MTH 301: Groups Theory (4)

***Learning Objectives*:**

This is an introductory course on Group theory. We will begin by studying the basic concepts of subgroups, homomorphisms and quotient groups with many examples. We then study group actions, and prove the Class equation and the Sylow theorems. They are in turn used to prove the structure theorem for finite abelian groups and to discuss the classification of groups of small order. We then turn to solvability, prove the Jordan-Holder theorem, and discuss nilpotent groups (if time permits).

***Course Contents*:**

- Definition of group, basic properties, examples (Dihedral, Symmetric, Groups of Matrices, Quaternion Group, Cyclic, Abelian Groups)
- Homomorphisms, Isomorphisms, subgroups, subgroup generated by a set, subgroups of cyclic groups
- Review of Equivalence relations, Cosets, Lagrange’s theorem, Normal subgroup, Quotient Group, Examples, Isomorphism theorems, Automorphisms
- Group actions, orbits, stabilizer, faithful and transitive actions, centralizer, normalizer, Cayley’s theorem, Action of the group on cosets
- Conjugation, Class equation, Cauchy’s theorem, Applications to p-groups, Conjugacy in S_n
- Sylow theorems, Simplicity of A_n and other applications
- Direct products, Structure of Finite abelian groups
- Semi-Direct products, Classification of groups of small order
- Normal series, Composition series, Solvable groups, Jordan-H¨older theorem, Insolvability of S_5
- Lower and upper central series, Nilpotent groups, Basic commutator identities, Decomposition theorem of finite nilpotent groups (if time permits)

***Suggested Books*:**

- I. N. Herstein, Topics in Algebra, 2nd Edition, Wiley, 2006
- T. W. Hungerford, Algebra, Springer Verlag, 2005
- M. Artin, Algebra, Prentice-Hall of India, 1994
- D. S. Dummit, R. M. Foote, Abstract Algebra, 2nd Edition, Wiley
- J. Rotman, A First Course in Abstract Algebra : With Applications, Prentice Hall
- J. Rotman, An Introduction to Theory of Groups, Springer GTM, 1999
- H. Kurzweil, B. Stellmacher, The Theory of Finite Groups, Springer Universitext, 2004
- M. Suzuki, Group Theory I, Springer GMW 247

MTH 302: Rings and Modules (4)

Pre-requisites: *MTH 301*

***Learning Objectives*:**

This is an introductory course on Group theory. We will begin by studying the basic concepts of subgroups, homomorphisms and quotient groups with many examples. We then study group actions, and prove the Class equation and the Sylow theorems. They are in turn used to prove the structure theorem for finite abelian groups and to discuss the classification of groups of small order. We then turn to solvability, prove the Jordan-Holder theorem, and discuss nilpotent groups (if time permits).

***Course Contents*:**

- Definition of rings, Homomorphisms, basic examples (Polynomial ring, Matrix ring, Group ring), Integral domain, field, Field of fractions of an integral domain
- Ideals, Prime and Maximal ideals, Quotient Rings, Isomorphism theorems, Chinese Remainder theorem, Applications
- Principal ideal domains, Irreducible elements, Unique factorization domains, Euclidean domains, examples
- Polynomial rings, ideals in polynomial rings, Polynomial rings over fields, Gauss' Lemma, Polynomial rings over UFDs, Irreducibility criteria
- Definition of modules, submodules, The group of homomorphisms, Quotient modules, Isomorphism theorems, Direct sums, Generating set, free modules, Simple modules, vector spaces
- Free modules over a PID, Finitely generated modules over PIDs
- Applications to finitely generated abelian groups and Rational and Jordan canonical forms
- (if time permits) Tensor product of modules, Exact sequences of modules, Homfunctor, Projective modules, Injective modules, Baer's criterion

***Suggested Books*:**

- D.S. Dummit, R.M. Foote, Abstract Algebra, 2nd Edition, Wiley
- G. Birkhoff, S. McLane, Algebra (3rd Edition), AMS
- S. Lang, Algebra (3rd Edition), Pears
- C. Musili, Rings and Modules (2nd Edition), Narosa
- M.F. Atiyah, I.G. Macdonald, Introduction to Commutative Algebra (1st Indian Edition), Levant Books
- N. Jacobson, Basic Algebra (Vols - I & II), Hindustan Book Agency

MTH 303: Real Analysis I (4)

***Learning Objectives*:**

This is an introductory course on analysis for BS-MS mathematics students. The aim of this course is to introduce and develop basic analytic concepts of limit, convergence, integration and differentiation. Students who have taken a first course in calculus are suitable for this course.

***Course Contents*:**

Real number system, limit superior, limit inferior, supremum principle, completeness, Cantor set

Sequences and series of functions, uniform convergence and its consequences, space of continuous functions on a closed interval, equicontinuous families, Stone-Weierstrass theorem, Arzela-Ascoli theorem

Taylor's theorem, power series, radius of convergence, exponential, trigonometric and logarithmic functions

Monotonic functions, functions of bounded variation, rectifiable curves

Riemann-Stieltjes integral, properties of Riemann-Stieltjes integral, differentiation of the integral, fundamental theorem of calculus, integration by parts, Gamma function

***Suggested Books*:**

- T. M. Apostol, *Calculus, Volumes 1 and 2* (2nd edition), Wiley Eastern, 1980
- W. Rudin, *Principles of Mathematical Analysis (3rd Edn.),* McGraw Hill, 1953
- T. M. Apostol, *Mathematical Analysis (2nd Edn.),* Narosa Publishing, 1985
- R. R. Goldberg, *Methods of Real Analysis*
- H. L. Royden, *Real Analysis (3rd Edn.),* Prentice Hall, 2008
- Terrance Tao, *Analysis I & II,* TRIM Series, Hindustan Book Agency

MTH 304: Metric Spaces and Topology (4)

Pre-requisites*: MTH 303 Real Analysis I*

Definition, open sets, closed sets, limit points, convergence, completeness, Baire's theorem, continuity, spaces of continuous functions

Compactness, sequential compactness, compact metric spaces, compact-open topology, Ascoli's theorem

Completeness, space filling curve, nowhere differentiable functions

Topology

Definition and examples of topology, base, subbase, weaker and stronger topology

Order topology, subspace topology, product and box topology

Continuity, homeomorphisms, quotient topology

Compact spaces, examples, Tychonoff's theorem and locally compact spaces, limit point compactness, local compactness

Connected spaces, components, path components, totally disconnected spaces, locally connected spaces, examples

Countability axioms, separation axioms, completely regular and normal spaces, Urysohn's lemma, Tietze extension theorem, Urysohn embedding theorem, Stone-Cechcompacitification

Suggested Books**:**

- G. F. Simmons, *Introduction to Topology and Modern Analysis*, Tata McGraw Hill, 2008
- J. R. Munkres, *Topology (2nd Edn)*, Dorling Kindersley, 2006

MTH 305: Elementary Number Theory (4)

Pre-requisites (recommended)*: MTH 101: Calculus of One Variable*

Learning Objectives**:**

The aim of this course is to develop a conceptual understanding of the elementary theory of numbers and to expose the students to writing proper mathematical proofs.

Course Contents**:**

Foundations: Principle of mathematical induction (with emphasis on writing a few basic proofs), binomial theorem, countable and uncountable sets, some basic results on countability, countability of Z, Q and uncountability of R.

Divisibility: Basic properties, division algorithm, GCD, LCM, properties of GCD, relation between GCD and LCM, Euclidean algorithm for finding GCD, Pythagorean triples, linear Diophantine equations, fundamental theorem of arithmetic, Euclid's lemma, existence of infinitely many primes.

Modular arithmetic: Basic properties of congruences, linear congruences, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem.

Number theoretic functions: Arithmetic functions (tau, sigma and Mobius) and their properties (specifically multiplicative property of the functions tau, sigma and the Mobius inversion formula), Euler's phi function and its properties, Euler's Theorem, Fermat's little theorem as a corollary of Euler's theorem.

Quadratic reciprocity: Primitive roots (order of an integer modulo n, primitive roots for primes), quadratic congruences, definition of quadratic residue, Legendre symbol and its properties, quadratic reciprocity law.

Continued fractions: Finite continued fractions, approximation of rational numbers by finite simple continued fractions, solution of linear Diophantine equations using finite continued fractions, infinite continued fractions, unique representation of irrationals as an infinite continued fraction, Pell's equation and its solutions using continued fractions.

Suggested Books**:**

Textbooks:

- David Burton, Elementary Number Theory, 7th edition, McGraw Hill Education, 2012.
- John Stillwell, Elements of Number Theory, 1st edition, Springer, 2003.

References:

- James Tattersall, Elementary Number Theory in Nine Chapters, 1st edition, Cambridge University Press, 1999.
- Ya. Khinchin, Continued Fractions, 3rd edition, Dover, 1997.
- Thomas Koshy, Elementary Number Theory with Applications, 2nd edition, Elsevier, 2007.

MTH 306: Ordinary Differential Equations (4)

Pre-requisites*: MTH 303 Real Analysis I*

First-Order Linear equations: exact equations, orthogonal trajectories, homogeneous equations, integrating factors, reduction of order

Second-order linear equations: equations with constant coefficients, method of undetermined coefficients, variation of parameters, power series solutions, special functions, applications

Higher-order linear equations

Some basic concepts of Fourier series

Quick review of elementary linear algebra, Picard's existence and uniqueness theorem, Sturm comparison theorem

Systems of first-order equations, homogeneous linear systems with constant coefficients

Non-linear equations: critical points and stability, Liapunov's direct method, Poincare-Bendixson theory

Suggested Books**:**

- George F. Simmons & Steven Krantz, *Differential equations*, Paperback edition, Tata-McGraw Hill 2009
- G. Birkhoff& G. C. Rota, *Ordinary differential equations*, Paperback edition, John Wiley &Sons, 1989
- E. Coddington& N. Levinson, *Theory of ordinary differential equations*, Paperback edition, Tata-McGrawa Hill, 2008
- W. Hurewicz, *Lectures on ordinary differential equations*, Dover, New York, 1999

MTH 307: Programming and Data Structure (4)

Programming in a structured language such as C

Data Structures: definition, operations, implementations and applications of basic data structures

Array, stack, queue, dequeue, priority queue, double linked list, orthogonal list, binary tree and traversal algorithm, threaded binary tree, generalized list

Binary search, Fibonacci search, binary search tree, height balance tree, heap, B-tree, digital search tree, hashing techniques

***Suggested Books*:**

- Donald E. Knuth, *The art of computer programming (*five volumes, 0 - 4*)*, Addison Wesley
- V. Aho, J. E. Hopcroft & J. E. Ullman, *Data Structures & Algorithm,* Addison Wesley
- W. Kernighan, D. M. Richie, *The C Programming Language*, Prentice Hall

MTH 308: Combinatorics and Graph Theory (4)

Combinatorics: Elementary principles of combinatorics (permutations and combinations), binomial coefficients, inclusion-exclusion principle, generating functions, recurrence relation, pigeon-hole principle and Ramsey theory

Graph theory: definition, isomorphisms, degree sequences, connectivity, trees, colourings, Eulerian graphs, directed graphs, network flows

***Suggested Books*:**

- R. A. Brualdi, *Introductory Combinatorics* (5th Ed.), Prentice Hall
- F. Harary, *Graph Theory*, Westview Press
- Bondy, U. S. R. Murty, *Graph Theory* (1st Ed.), Springer, GTM
- S. M. Cioaba& M. Ram Murty, *A First Course in Graph Theory*, TRIM Series, HBA

MTH 310: Introduction to probability theory (4)

Prerequisites: NIL

Course contents:

Probability: Classical, relative frequency and axiomatic definitions of probability, addition rule and conditional probability, total probability, Bayes Theorem and independence, equally likely experiments, coin tossing and random walk. Random Variables: Discrete, continuous and mixed random variables, probability mass, probability density and cumulative distribution functions, mathematical expectation, moments, probability and moment generating function, median and quantiles, Markov inequality, Chebyshev'sinequality,weak law of large numbers and central limit theorem. Special Distributions (Binomial, Poisson, Normal), Joint Distributions: Joint, marginal and conditional distributions. Joint distributions of independent random variables and applications to find the sum, product and ratio of random variables. Transformations, generating functions, convolution and its connection with probability distributions.
Random walk: Reflection principle.
Markov chain: Connection with random walk. Recurrence and transience.
Stationary distribution (if time permits).

***Suggested Books*:**

- W. Feller: Introduction to the Theory of Probability and its Applications, (Vols. 1 & 2).
- K. L. Chung: Elementary Probability Theory.
- S. M. Ross: A First Course in Probability.

***References*:**

- R. Ash: Basic Probability Theory.
- P. G. Hoel, S. C. Port and C. J. Stone: Introduction to Probability Theory.

MTH 311: Advanced Linear Algebra (4)

***Learning Objectives*:**

This course reviews undergraduate linear algebra and proceeds to more advanced topics. Its purpose is to provide a solid understanding of linear algebra of the sort needed throughout graduate mathematics.

***Course Contents*:**

Vector spaces, subspaces, bases and dimension, some examples.

Linear transformations, rank - nullity theorem, the algebra of linear transformations, invertible linear transformations, matrix of a linear transformation, change of basis, linear functionals, annihilator of a subspace, dual space, double dual, canonical isomorphism between a vector space and its double dual, transpose of a linear transformation.

Characteristic values, diagonalizable linear operator, equivalent notions of diagonalizable operator (in terms of characteristic polynomial, dimensions of eigen spaces), annihilating polynomials, minimal polynomial, characterization of diagonalizable operator using the minimal polynomial, invariant subspaces, simultaneous triangulation, simultaneous diagonalization, direct sum decompositions, projections, invariant direct sums, primary decomposition theorem, nilpotent operators, S-N decomposition.

Rational and Jordan forms.
Inner product space, Gram-Schmidt, linear functionals and adjoints, unitary operators, normal operators, self-adjoint operators, spectral theorem for self-adjoint operators.

***Suggested Books*:**

- K. Hoffman and R. Kunze, *Linear Algebra*, Prentice-Hall, 1961
- Serge Lang, *Linear Algebra (2nd Edition)*, Addition-Wesley Publishing, 1971
- M.W. Hirsch and S. Smale, Differential equations, dynamical systems and linear algebra, Pure and Applied Mathematics, Vol. 60, Academic Press, 1974
- P. Halmos, Finite dimensional vector spaces (2nd Edition), Undergraduate texts in Mathematics, Springer-Verlag New York Inc., 1987
- Serge Lang, Algebra, Graduate Texts in Mathematics (3rd Edition), Springer-Verlag New York Inc., 2005

MTH 314: Probability and Statistics (4)

Course contents:

Algebra of Sets: sets, classes, limit of a sequence of sets, rings, sigma rings, fields, sigma-fields, monotone classes.

Probability: Classical, relative frequency and axiomatic definitions of probability, addition rule and conditional probability, multiplication rule, total probability, Bayes Theorem and independence, problems.

Random Variables: Discrete, continuous and mixed random variables, probability mass, probability density and cumulative distribution functions, mathematical expectation, moments, probability and moment generating function, median and quantiles, Markov inequality, Chebyshev's inequality, problems.

Special Distributions, Joint Distributions: Joint, marginal and conditional distributions, product moments, correlation and regression, independence of random variables, bivariate normal distribution.

Transformations: functions of random vectors, distributions of order statistics, distributions of sums of random variables.

Sampling Distributions: The Central Limit Theorem, distributions of the sample mean and the sample variance for a normal population, Chi-Square, t and F distributions.

Descriptive Statistics: Graphical representation, Summarization and tabulation of data.

Estimation: Unbiasedness, consistency, the method of moments and the method of maximum likelihood estimation, confidence intervals for parameters in one sample and two sample problems of normal populations, confidence intervals for proportions.

Testing of Hypotheses: Null and alternative hypotheses, the critical and acceptance regions, two types of error, power of the test, the most powerful test and Neyman-Pearson Fundamental Lemma, tests for one sample and two sample problems for normal populations, tests for proportions, Chi-square goodness of fit test and its applications.

***Suggested Books*:**

- W. Feller, An Introduction to Probability Theory and Its Applications, Volume 1, 3rd Edition, Wiley, 1968
- V. Rohatgi, A. Saleh, Introduction to Probability Theory and Statistics, 2nd Edition, Wiley, 2000
- S.M. Ross, A First Course in Probability, 6th Edition, Prentice Hall
- A. Craig, R. Hogg, J. McKean, Introduction to Mathematical Statistics, 6th Edition, Prentice Hall, 2004
- J.S. Milton and J.C. Arnold, Introduction to Probability and Statistics
- P. Hoel, S. Port, C. Stone, Introduction to Probability Theory, 1st Edition, Brooks Cole, 1972

MTH 401: Fields and Galois Theory (4)

Pre-requisites*: MTH 301 Groups and Rings*

Learning Objectives**:**

Field Extensions are studied in an attempt to find a formula for the roots of polynomial equations, similar to the one that exists for a quadratic equation. The Galois group is introduced as a way to capture the symmetry between these roots; and the solvability of the Galois group determines if such a formula exists or not. In the 19th century, Galois proved that a formula does not exist for a general 5th degree equation. More importantly, the use of groups to study the symmetry of other objects is a pervasive theme in Mathematics, and this is traditionally the first place where one encounters it.
The topics to be covered include irreducibility of polynomials, Field Extensions, Normal and Separable Extensions, Solvable Groups, and Solvability of polynomial equations by radicals, Finite fields, and Cyclotomic fields

Course Contents**:**

Polynomial rings, Gauss lemma, Irreducibility criteria

Definition of a field and basic examples, Field extensions

Algebraic extensions and algebraic closures

Classical Straight hedge and compass constructions (optional)

Splitting fields, Separable and Inseparable extensions

Cyclotomic polynomials, Galois extensions

Fundamental theorem of Galois theory

Composite and Simple extensions, Abelian extensions over Q

Galois groups of polynomials, Solvability of groups, Solvability of polynomials

Computations of Galois groups over Q

Suggested Books**:**

- D. S. Dummit, R. M. Foote, *Abstract Algebra* (2nd Ed.), Wiley
- S. Lang, *Algebra* (3rd Ed.), Pears

MTH 403: Real Analysis II (4)

***Pre-requisites**: MTH 303 Real Analysis I*

***Learning Objectives*:**

This course deals with the study of functions of several real variables and the geometry associated with such functions.
There are two parts to this course. The first part deals with the study of differentiation and integration of such functions. The second part is devoted to the statement and proof of the higher dimensional version of the fundamental theorem of calculus, viz, Stoke's theorem (and its companions).
This is one of the standard courses in any mathematics curriculum. It also serves as a first introduction to differential geometry and topology.

***Course Contents*:**

Vector-valued functions, continuity, linear transformations, differentiation, total derivative, chain rule

Determinants, Jacobian, implicit function theorem, inverse function theorem, rank theorem

Partition of unity, Derivatives of higher order

Riemann integration in $\mathbf{R}^n$, differentiation of integrals, change of variables, Fubini's theorem

Exterior algebra, simplices, chains of simplices, Stokes theorem, vector fields, divergence of a vector field, Divergence theorem, closed and exact forms, Poincare lemma

***Suggested Books*:**

- David Widder, *Advanced Calculus*, second edition, Dover, 1989
- M. Spivak, *Calculus on manifolds*, fifth edition, Westview Press, 1971
- J. Munkres, *Elementary Differential topology*, Princeton University Press, 1966

MTH 404: Measure and Integration (4)

***Pre-requisites**: MTH 403 Real Analysis II*

Topology of the real line, Borel, Hausdorff and Lebesgue measures on the real line, regularity properties, Cantor function

σ-algebras, measure spaces, measurable functions, integrability, Fatou's lemma, Lebesgue's monotone convergence theorem, Lebesgue's dominated convergence theorem, Egoroff's theorem, Lusin's theorem, the dual space of C(**X**) for a compact, Hausdorff space, **X**

Comparison with Riemann integral, improper integrals

Lebesgue's theorem on differentiation of monotonic functions, functions of bounded variation, absolute continuity, differentiation of the integral, Vitali's covering lemma, fundamental theorem of calculus

Holder's, inequality, Minkowski's inequality, convex functions, Jensen's inequality, $\mathbf{L}^p$ spaces, Riesz-Fischer theorem, dual of $\mathbf{L}^p$ spaces

***Suggested Books*:**

- W. Rudin, *Real and Complex Analysis*, third edition. Tata-McGraw Hill, 1987
- H. Royden, *Real Analysis*, third edition, Prentice-Hall of India, 2008
- R. Wheeden, A. Zygmund, *Measure and Integral*, Taylor and Francis, 1977
- J. Kelley, T. Srinivasan, *Measure and Integral*, Volume I, Springer, 1987
- Rana, *An Introduction to Measures and Integration*, Narosa Publishing House
- E. Lieb, M. Loss, *Analysis*, Narosa Publishing House

MTH 405: Partial Differential Equations (4)

***Pre-requisites**: MTH 306 Ordinary Differential Equations*

***Learning Objectives*:**

This is an introductory course in partial differential equations for students majoring in mathematics. After discussing the solutions of first-order linear and quasi-linear equations in considerable detail we introduce the Cauchy problem for first and higher order equations and then briefly discuss the Cauchy-Kovalevski existence theorem and Holmgren's uniqueness theorem. We follow this by a study of second-order linear equations; here the goal is to understand the solutions of the three prototypical equations, Laplace, Wave and the Heat equation, in the classical set-up.

***Course Contents*:**

First-order equations: linear and quasi-linear equations, general first-order equation for a function of two variables, Cauchy problem, envelopes

Higher-order equations: Cauchy problem, characteristic manifolds, real analytic functions, Cauchy-Kovalevski theorem, Holmgren’s uniqueness theorem

Laplace equation: Green’s identity, Fundamental solutions, Poisson’s equation, Maximum principle, Dirichlet problem, Green’s function, Poisson’s formula

Wave equation: spherical means, Hadamard’s method, Duhamel’s principle, the general Cauchy problem

Heat equation: initial-value problem, maximum principle, uniqueness, regularity

***Suggested Books*:**

- F. John, *Partial differential equations*, 4th edition, Springer, 1982
- G. B. Folland, *Introduction to Partial differential equations*, 2nd edition, Princeton University Press, 1995
- J. Rauch, *Partial differential equations*, Springer, GTM 128, 1991
- L. Evans, *Partial differential equations,* American Mathematical Society GSM series, 1998

MTH 406: Differential Geometry of Curves and Surfaces (4)

***Pre-requisites**: MTH 306 Ordinary Differential Equations*

Curves: curves in space, tangent vector, arc length, curvature, torsion, Frenet formulas

Surfaces: parametrization, tangent plane, orientability, first fundamental form, area, orientation, Gauss map, second fundamental form, Gauss curvature, ruled and minimal surfaces

Geodesics, isometries of surfaces, Gauss' TheoremaEgregium, Codazzi-Mainardi equations

Gauss-Bonnet theorem for compact surfaces

***Suggested Books*:**

- Pressley, *Elementary Differential Geometry*, Springer, Indian reprint, 2004
- Manfredo do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice Hall, 1976
- D. J. Struik, *Lectures on Differential Geometry*, Dover, 1988
- Barrett O'Neill, *Elementary Differential Geometry*, Second edition, Academic Press (Elsevier), 2006

MTH 407: Complex Analysis I (4)

Pre-requisites (Desirable)*: MTH 303: Real Analysis I*

Learning Objectives**:**

The aim of this course is to introduce the theory of modular forms with minimal prerequisites. The learning objectives of this course include the definition of analyticity, the Cauchy-Riemann equations and the concept of differentiability. Also to be learnt are the theorems on entire functions, residue theorem and applications and finally conformal mapping.

Course Contents**:**

- Complex numbers: powers and roots, geometric representation, stereographic projection
- Complex differentiability: limits, continuity and differentiability, Cauchy Riemann equations, definition of a holomorphic function
- Elementary functions: sequences and series, complex exponential, trigonometric, and hyperbolic functions, the logarithm function, complex powers, Mobius transformations
- Complex integration: contour integrals, Cauchy's integral theorem in a disc, Cauchy’s Integral Formula, Liouville’s theorem, Fundamental Theorem of Algebra, Morera’s theorem, Schwarz reflection principle
- Series representation of analytic functions: Taylor series, power series, zeros and singularities, Laurent decomposition, open mapping theorem, Maximum Principle
- Residue theory: residue formula, calculation of certain improper integrals, Riemann’s theorem on removable singularities, CasoratiWeierstrass theorem, the argument principle and Rouche's theorem
- Conformal mappings: conformal maps, Schwarz lemma and automorphisms of the disk and the upper half plane

Suggested Books**:**

Texts

- Elias M. Stein, Rami Shakarchi, Complex Analysis, Princeton University Press, 2003
- Theodore W. Gamelin, Complex Analysis, Springer Verlag, 2001
- John B. Conway, Functions of one Complex Variable I, Springer, 1978
- E. Freitag and R.Busam, Complex Analysis, Springer, 2005

References

- Lars Ahlfors, Complex Analysis. McGrawHill, 1979
- R. Remmert, Theory of Complex Functions. Springer Verlag, 1991
- C. Caratheodory, Theory of Functions of a complex variable, AMS Chelsea, 2001

MTH 408: Numerical Analysis (4)

***Pre-requisites**: MTH 303 Real Analysis I*

Round off errors and computer arithmetic

Interpolation: Lagrange interpolation, divided differences, Hermite interpolation, splines, numerical differentiation, Richardson extrapolation

Numerical Integration: trapezoidal, Simpsons, Newton-Cotes, Gauss quadrature, Romberg integration, multiple integrals

Solution of linear algebraic equations: direct methods, Gauss elimination, pivoting, matrix factorizations

Iterative methods: matrix norms, Jacobi and Gauss-Siedel methods, relaxation methods

Computation of eigenvalues and eigenvectors: power method, householders method, QR algorithm

Numerical solutions of non-linear algebraic equations: bisection, secan and Newton's, zeroes of polynomials

***Suggested Books*:**

- R. L. Burden, D. J. Faires, *Numerical Analysis*
- E. K. Blum, *Numerical Analysis and Computation*, *Theory and Practice,* Dover, 2010
- S. D. Conte, C. De Boor, *Elementary Numerical Analysis*,third edition, McGraw-Hill, 1980
- D. M. Young, R. T. Gregory, *A Survey of Numerical Mathematics*, volumes 1 and 2, Dover, 1988

MTH 409: Optimization Techniques (4)

***Pre-requisites**: MTH 303 Real Analysis I*

Maxima and minima, Lagrange multipliers method, formulation of optimization problems, linear programming, non-linear programming, integer programming problems

Convex sets, separating hyperplanes theorem, simplex method, two phase simplex method, duality theorem, zero-sum two-person games, branch and bound method of integer linear programming

Dynamic programming, Bellman’s principle of optimality

***Suggested Books*:**

- Katta G. Murty, *Linear Programming*, Revised edition, Wiley, 1983
- Griva, S. Nash, A. Sofer, *Linear and Non-linear Optimization*, second edition, SIAM, 2008
- M. Bazaraa, H. Sherali, C. Shetty, *Non-linear Programming: Theory and Algorithms*, third edition, Wiley Inter-Science, 2006

MTH 410: Representation Theory (4)

***Pre-requisites**: MTH 301, MTH 302*

Representations of groups, subrepresentations, Irreducible representations, tensor product of representations, Maschke's theorem, Wedderburn decomposition

Characters of representations, Generalized characters, Schurs lemma, Orthogonality, Regular representations, Decomposition theorems

Representations of direct product of finite groups, Induced representations, Reciprocity theorem

Representations and characters of standard finite and infinite groups: cyclic groups, dihedral groups, symmetric and alternating groups of small order etc.

Applications of Representation Theory

***Suggested Books*:**

- J. P. Serre, *Linear Representations of Finite groups* (Graduate Texts in Mathematics), 2nd Edition, Springer-Verlag New York Inc., 1977
- W. Fulton and J. Harris, *Representation Theory*, *A First Course*, 2nd Indian reprint, Springer India, 2007
- *G. James, M. Liebeck, Representations and Characters of Groups*, Cambridge University Press, 2001

MTH 411: Introduction to Lie Groups and Lie Algebras (4)

***Pre-requisites**: Required : MTH 311, Desirable : MTH 301* Groups and Rings

***Learning Objectives*:**

The proposed course aims at providing and introduction to Lie groups, Lie algebras and their representations. The first part of the course focuses on matrix Lie groups (closed subgroups of GL(n; C)) and Lie algebras. The second part of the course deals with representations of semisimple Lie groups and Lie algebras. We begin with SU(2) and SU(3), as these cases very well illustrate the ideas of Cartansubalgebras, the roots, weights and the Wey1 group. We also look at Semisimple Lie groups and Lie algebras in general towards the end.

***Course Contents*:**

Matrix Lie Groups: Definition and examples; Lie group homomorphisms and isomorphisms, Lie subgroups, polar decomposition.

Lie algebras: matrix exponential and matrix logarithm (4), one parameter subgroups, the Lie algebra of a matrix Lie group, Lie subalgebras, complexification of a real Lie algebra

Banker-Campbell-Hausborffformula : Definition, computations on Heisenberg group, Integral form of B-C-H formula (no proof), the series form of the B-C-H formula (no proof), applications to exponential map

Representations of SU(3): Weights and roots, theorem of the highest weight, the Wey1 group, weight diagrams

Semisimple Lie algebras: Complete reducibility, examples of reductive and semisimple Lie algebras, Cartansubalgebras, roots and root spaces, inner products of roots and co-roots, the Wey1 group, root systems, positive roots, the example of sl(n,C) in detail, uniqueness results.

***Suggested Books*:**

- Hall, Brian Lie Groups, *Lie Algebras, and Representations. Graduate Texts in Mathematics*, Vol. 222, Springer Verlag, 2003.
- Rossmann, Wulf. *Lie Groups: An Introduction through Linear Groups*. Oxford Graduate Texts in Mathematics 5, Oxford University Press, 2002.
- Humphreys, James E. *Introduction to Lie Alogebras and Representation Theory. Graduate Texts in Mathematics,* Vol. 9, Springer, 1973.
- Baker, Andrew. *Matrix Groups: An Introduction to Lie Group Theory*. Springer Veriag, 2002.

MTH 412: Combinatorics and Graph Theory (4)

Combinatorics: Elementary principles of combinatorics (permutations and combinations), binomial coefficients, inclusion-exclusion principle, generating functions, recurrence relation, pigeon-hole principle and Ramsey theory

Graph theory: definition, isomorphisms, degree sequences, connectivity, trees, colourings, Eulerian graphs, directed graphs, network flows

***Suggested Books*:**

- R. A. Brualdi, *Introductory Combinatorics* (5th Ed.), Prentice Hall
- F. Harary, *Graph Theory*, Westview Press
- Bondy, U. S. R. Murty, *Graph Theory* (1st Ed.), Springer, GTM
- S. M. Cioaba& M. Ram Murty, *A First Course in Graph Theory*, TRIM Series, HBA

MTH 414: Probability and Statistics (4)

Course contents:

Algebra of Sets: sets, classes, limit of a sequence of sets, rings, sigma rings, fields, sigma-fields, monotone classes.

Probability: Classical, relative frequency and axiomatic definitions of probability, addition rule and conditional probability, multiplication rule, total probability, Bayes Theorem and independence, problems.

Random Variables: Discrete, continuous and mixed random variables, probability mass, probability density and cumulative distribution functions, mathematical expectation, moments, probability and moment generating function, median and quantiles, Markov inequality, Chebyshev's inequality, problems.

Special Distributions, Joint Distributions: Joint, marginal and conditional distributions, product moments, correlation and regression, independence of random variables, bivariate normal distribution.

Transformations: functions of random vectors, distributions of order statistics, distributions of sums of random variables.

Sampling Distributions: The Central Limit Theorem, distributions of the sample mean and the sample variance for a normal population, Chi-Square, t and F distributions.

Descriptive Statistics: Graphical representation, Summarization and tabulation of data.

Estimation: Unbiasedness, consistency, the method of moments and the method of maximum likelihood estimation, confidence intervals for parameters in one sample and two sample problems of normal populations, confidence intervals for proportions.

Testing of Hypotheses: Null and alternative hypotheses, the critical and acceptance regions, two types of error, power of the test, the most powerful test and Neyman-Pearson Fundamental Lemma, tests for one sample and two sample problems for normal populations, tests for proportions, Chi-square goodness of fit test and its applications.

***Suggested Books*:**

- W. Feller, An Introduction to Probability Theory and Its Applications, Volume 1, 3rd Edition, Wiley, 1968
- V. Rohatgi, A. Saleh, Introduction to Probability Theory and Statistics, 2nd Edition, Wiley, 2000
- S.M. Ross, A First Course in Probability, 6th Edition, Prentice Hall
- A. Craig, R. Hogg, J. McKean, Introduction to Mathematical Statistics, 6th Edition, Prentice Hall, 2004
- J.S. Milton and J.C. Arnold, Introduction to Probability and Statistics
- P. Hoel, S. Port, C. Stone, Introduction to Probability Theory, 1st Edition, Brooks Cole, 1972

MTH 415: Commutative Algebra (4)

***Pre-requisites**: MTH 401 and its pre-requisites*

Quotient Rings, Prime and Maximal ideals, units, Nilradical, Jacobson Radical, Operations on ideals, Extensions and contractions

Tensor product of Algebras (only existence theorem), Rings and Modules of fractions, Local properties, Structure passing between R and $S^{-1}R$ (resp. M and $S^{-1}M$)

Primary decompositions, Uniqueness theorems, Chain conditions, Noetherian and Artinian Rings, Lasker-Noether theorem, Hilbert basis theorem, Nakayama's lemma, Krull intersection theorem

Integral dependence, Going up theorem, Integrally closed integral domains, Going down theorem

Valuation rings, Discrete valuation rings, Dedekind domains, Fractional ideals

Valuations, Completions, Extensions of absolute values, residue field, Local fields, Ostrowski's theorem

Hilbert's Nullstellensatz

***Suggested Books*:**

- Introduction to Commutative Algebra, Atiyah, M and Macdonald, I.G., Levant Books, Kolkata
- Graduate Algebra: Commutative View, Rowen, L.H., Graduate Studies in Mathematics, AMS
- Commutative Algebra with a view towards Algebraic Geometry, Eisenbud, D., Springer

MTH 417: Programming and Data Structure (4)

Programming in a structured language such as C

Data Structures: definition, operations, implementations and applications of basic data structures

Array, stack, queue, dequeue, priority queue, double linked list, orthogonal list, binary tree and traversal algorithm, threaded binary tree, generalized list

Binary search, Fibonacci search, binary search tree, height balance tree, heap, B-tree, digital search tree, hashing techniques

***Suggested Books*:**

- Donald E. Knuth, *The art of computer programming (*five volumes, 0 - 4*)*, Addison Wesley
- V. Aho, J. E. Hopcroft & J. E. Ullman, *Data Structures & Algorithm,* Addison Wesley
- W. Kernighan, D. M. Richie, *The C Programming Language*, Prentice Hall

MTH 503: Functional Analysis (4)

Pre-requisites (Desirable)*: MTH 404: Measure and Integration*

Learning Objectives**:**

Functional analysis is the branch of mathematics concerned with the study of spaces of functions. This course is intended to introduce the student to the basic concepts and theorems of functional analysis with special emphasis on Hilbert and Banach Space Theory. This gives the basics for more advanced studies in modern Functional Analysis, in particular in Operator Algebra Theory and Banach Space Theory.

Course Contents**:**

- Normed Linear spaces, Bounded Linear Operators, Banach Spaces, Finite dimensional spaces, Quotient Spaces
- Hilbert spaces, Riesz Representation Theorem, Orthonormal sets, Bessel's Inequality, Parseval's Identity, Fourier Series
- Dual Spaces, Dual of L^p spaces , Hahn-Banach Extension Theorem, Applications
- Open Mapping Theorem, Closed Graph Theorem, Uniform Boundedness Principle
- Weak and Weak-* topologies, Hahn-Banach Separation Theorem, Alaoglu's Theorem, Reflexivity
- Compact Operators, Adjoint of an operator, Spectral theorem for Compact Self-Adjoint operators
- (If time permits) Banach Algebras, Ideals and Quotients, Gelfand-Mazur Theorem, Fredholm Alternative, Fredholm Operators, Atkinson's theorem

Suggested Books**:**

- J.B. Conway, A Course in Functional Analysis, 2nd Ed., (Springer-Verlag, 1990)
- S. Kesavan, Functional Analysis, TRIM 52, Hindustan Book Agency
- B.V. Limaye, Functional Analysis, 2nd Ed., (New Age International, 1996)
- Martin Schechter, Principles of Functional Analysis, 2nd Ed., Graduate Studies in Mathematics, AMS
- P.D Lax, Functional Analysis, (Wiley, 2002)
- W. Rudin, Functional Analysis, 2nd Ed., (Tata McGraw-Hill, 2006)

MTH 504: Complex Analysis II (4)

***Pre-requisites**:*
Required: MTH 303 Real Analysis I, MTH 407 Complex Analysis
Desirable: MTH 304 Metric Spaces and Topology, MTH 503 Functional Analysis

Review of elementary concepts: Complex differentiation, Cauchy-Riemann equations, holomorphicity, complex integration, Cauchy's theorem and Cauchy's integral formula, Taylor and Laurent series, residue theorem, definition of a meromorphic function.

Harmonic functions: definition and properties, Poisson integral formula, mean-value property, Schwarz reflection principle, Dirichlet problem

Maximum modulus principle: Maximum modulus theorem, Schwarz lemma, Phragmen-Lindelof theorem

Approximations by rational functions: Runge's theorem, Mittag-Leffler theorem

Conformal mappings: definition and examples, space of holomorphic functions, Montel's theorem, statement of Riemann mapping theorem

Entire functions, Infinite products, Weierstrass factorization theorem, little and big Picard Theorems, Gamma function

***Suggested Books*:**

Texts:

- Stein E.M. and Shakarchi R., *Complex Analysis (Princeton Lectures in Analysis Series, Vol. II)*, Princeton University Press, 2003
- Conway J.B., *Functions of One Complex Variable*, Springer-Verlag NY, 1978
- Rudin W., *Real and Complex Analysis*, McGraw-Hill, 2006
- Epstein B. and Hahn L-S., Classical Complex Analysis, Jones and Bartlett, 2011
- Ahlfors L., *Complex Analysis*, Lars Ahlfors, McGraw-Hill, 1979.

References:

- Carathodory C., *Theory of functions of a complex variable, Vol. I and II*, Chelsea Pub Co, NY 1954
- Remmert R., Classical topics in complex function theory, Springer 1997

MTH 505: Introduction to Ergodic Theory (4)

***Pre-requisites**: MTH 304 Metric Spaces and Topology, MTH 404 Measure and Integration*

Discrete Dynamical systems: definition and examples - maps on the circle, the doubling map, shifts of finite type, toralautomorphisms.

Topological and Symbolic dynamics: transitivity, minimality, topological conjugacy and discrete spectrum, topological mixing, topological entropy, topological dynamical properties of shift spaces, circle maps and rotation number.

Ergodic Theory: invariant measures and measure-preserving transformations, ergodicity, recurrence and ergodic theorems (Poincare recurrence, Kac's lemma, Von Neumann's ergodic theorem, Birkhoff's ergodic theorem), applications of the ergodic theorem (continued fractions, Borel normal numbers, Khintchine’s recurrence theorem), ergodic measures for continuous transformations and their existence, Weyl’s equidistribution theorem, mixing and spectral properties.

Information and entropy - topological, measure-theoretic, and their relationship. Skew products, factors and natural extensions, induced transformations, suspensions and towers. Topological pressure and the variational principle, thermodynamic formalism and transfer operators, applications of thermodynamic formalism: (i) Bowen's formula for Hausdorff dimension, (ii) central limit theorems.

***Suggested Books*:**

- P. Walters, An Introduction to Ergodic Theory, Springer-Verlag, New York, 1982
- M.G. Nadkarni, Basic Ergodic Theory, Second Edition, Hindustan Book Agency, India
- M. Brin and G. Stuck, Introduction to Dynamical Systems, CUP, 2002
- M. Pollicott and M. Yuri, Dynamical systems and Ergodic theory, CUP, 1998
- P. R. Halmos, Lectures on Ergodic Theory, Chelsea, New York, 1956
- W. Parry, B. Bollobas, W. Fulton, Topics in Ergodic Theory, CUP, 2004
- A.B. Katok and B. Hasselblatt, Introduction to the Modern Theory of Dynamical Systems, Cambridge, 1995

MTH 506: Fourier Analysis on the Real Line (4)

Pre-requisites*: MTH 404 Measure and Integration, MTH 503 Functional Analysis: Normed linear spaces, completeness, Uniform boundedness principle, MTH 405 Partial Differential Equations: Basic knowledge of Laplacian, Heat and Wave equations*

The vibrating string, derivation and solution to the wave equation, The heat equation

Definition of Fourier series and Fourier coefficients, Uniqueness, Convolutions, good kernels, Cesaro/Abel means, Poisson Kernel and Dirichlet's problem in the unit disc

Mean-square convergence of Fourier Series, Riemann-Lebesgue Lemma, A continuous function with diverging Fourier Series

Applications of Fourier Series : The isoperimetric inequality, Weyl's equidistribution Theorem, A continuous nowhere-differentiable function, The heat equation on the circle

Schwartz space*, Distributions*, The Fourier transform on **R**: Elementary theory and definition, Fourier inversion, Plancherel formula, Poisson summation formula, Paley-Weiner Theorem*, Heisenberg Uncertainty principle, Heat kernels, Poisson Kernels

(If time permits/possible project topic) Definition of Fourier transform on $\mathbf{R}^d$, Definition of X-ray transform in $\mathbf{R}^2$ and Radon transform in $\mathbf{R}^3$, Connection with Fourier Transform, Uniqueness

Suggested Books**:**

Texts:

- E.M.Stein and R. Shakarchi, *Fourier Analysis: An Introduction, Princeton Univ Press*, 2003
- (For topics marked with a*) W. Rudin, *Functional Analysis,* 2nd Ed, Tata McGraw-Hill, 2006

References:

- J. Douandikoetxea*, Fourier Analysis* (Graduate Studies in Mathematics), AMS, 2000
- L. Grafakos, *Classical Fourier Analysis* (Graduate Texts in Mathematics), 2nd Ed, Springer, 2008

MTH 507: Introduction to Algebraic Topology (4)

Pre-requisites*: MTH 301 Groups and Rings, MTH 304 Metric Spaces and Topology*

Suggested Books**:**

This is a first course in algebraic topology. The subject revolves around finding and computing invariants associated with topological spaces. The first such invariant is the fundamental group of a pointed topological space which we'll study in detail along with the classification of covering spaces using fundamental group actions.

Suggested Books**:**

The Fundamental Group: Homotopy, Fundamental Group, Introduction to Covering Spaces, The Fundamental Group of the circle $\mathbf{S}^1$, Retractions and fixed points, Application to the Fundamental Theorem of Algebra, The Borsuk-Ulam Theorem, Homotopy Equivalence and Deformation Retractions, Fundamental group of a product of spaces, and Fundamental group the torus $\mathbf{T}^2=\mathbf{S}^1\times\mathbf{S}^1$, n-sphere $\mathbf{S}^n$, and the real projective n-space $\mathbf{RP}^n$.

Van Kampen's Theorem: Free Products of Groups, The Van Kampen Theorem, Fundamental Group of a Wedge of Circles, Definition and construction of Cell Complexes, Application to Van Kampen Theorem to Cell Complexes, Statement of the Classification Theorem for Surfaces, and Fundamental groups of the closed orientable and non-orientable surfaces of genus g.

Covering Spaces: Universal Cover and its existence, Unique Lifting Property, Galois Correspondence of covering spaces and their Fundamental Groups, Representing Covering Spaces by Permutations – Deck Transformations, Group Actions, Covering Space Actions, Normal or Regular Covering Spaces, and Application of Covering Spaces to Cayley Complexes.

Suggested Books**:**

- J. R. Munkres, Topology (2nd Edition), Pearson Publishing Inc, 2000
- Hatcher, Algebraic Topology, Cambridge University Press, 2002
- M. A. Amstrong, Basic Topology, Springer International Edition, 2004
- W. S. Massey, Algebraic Topology: An Introduction , Springer, 1977
- J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988
- M. J. Greenberg and J. R. Harper, Algebraic Topology: A First Course, Perseus Books Publishing, 1981
- E. H. Spanier, Algebraic Topology, Springer, 1994

MTH 508: Introduction to Differentiable Manifolds and Lie Groups (4)

***Pre-requisites**: MTH 303 Real Analysis I, MTH 304 Metric Spaces and Topology, MTH 306 Ordinary Differential Equations, MTH 403 Real Analysis II*

Differentiable manifolds: definition and examples, differentiable functions, existence of partitions of unity, tangent vectors and tangent space at a point, tangent bundle, differential of a smooth map, inverse function theorem, implicit function theorem, immersions, submanifolds, submersions, Sard's theorem, Whitney embedding theorem

Vector fields: vector fields, statement of the existence theorem for ordinary differential equations, one parameter and local one-parameter groups acting on a manifold, the Lie derivative and the Lie algebra of vector fields, distributions and the Frobenius theorem

Lie groups: definition and examples, action of a Lie group on a manifold, definition of Lie algebra, the exponential map, Lie subgroups and closed subgroups, homogeneous manifolds: definition and examples

Tensor fields and differential forms: cotangent vectors and the cotangent space at a point, cotangent bundle, covector fields or 1-forms on a manifold, tensors on a vector space, tensor product, symmetric and alternating tensors, the exterior algebra, tensor fields and differential forms on a manifold, the exterior algebra on a manifold

Integration: orientation of a manifold, a quick review of Riemann integration in Euclidean spaces, differentiable simplex in a manifold, singular chains, integration of forms over singular chains in a manifold, manifolds with boundary, integration of n-forms over regular domains in an oriented manifold of dimension n, Stokes theorem, definition of de Rhamcohomology of a manifold, statement of de Rham theorem, Poincare lemma

***Suggested Books*:**

Texts:

- J. Lee, *Introduction to smooth manifolds*, Springer, 2002
- W. Boothby, *An Introduction to differentiable manifolds and Riemannian geometry,* Academic Press, 2002
- F. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer, GTM 94, 1983
- M. Spivak, *A comprehensive introduction to differential geometry, Vol. 1*, Publish or Perish, 1999

References:

- G. de Rham, *Differentiable manifolds: forms, currents and harmonic forms*, Springer, 1984
- V. Guillemin and A. Pollack., *Differential topology,* AMS Chelsea, 2010
- J. Milnor, *Topology from the differentiable viewpoint,* Princeton University Press, 1997
- J. Munkres, *Analysis on manifolds,* Westview Press, 1997
- C. Chevalley, *Theory of Lie groups,* Princeton University Press, 1999
- R. Abraham, J. Marsden, T. Ratiu, *Manifolds, tensor analysis, and applications*, Springer, 1988

MTH 509: Sturm-Liouville Theory (4)

***Pre-requisites**: MTH 306 Ordinary Differential Equations, MTH 404 Measure and Integration*

Fourier Series: Fourier series of a periodic function, question of point-wise convergence of such a series, behavior of the Fourier series under the operation of differentiation and integration , sufficient conditions for uniform and absolute convergence of a Fourier series, Fourier series on intervals, examples of boundary value problems for the one dimensional heat and wave equations illustrating the use of Fourier series in solving them by separating variables, a brief discussion on Cesarosummability and Gibbs phenomenon

Orthogonal Expansions: A quick review of L2 spaces on an interval, convergence, completeness, orthonormal systems, Bessel's inequality, Parseval's identity, dominated convergence theorem
Sturm-Liouville Systems: linear differential operators, formal adjoint of a linear operator, Lagrange's identity, self-adjoint operators, regular and singular Sturm-Liouville systems, Sturm-Liouville series, Prufer substitution, Sturm comparison and oscillation theorems, eigenfunctions, Liouville normal form, distribution of eigenvalues, normalized eigenfunctions, Green's functions, completeness of eigenfunctions

Illustrative boundary value problems: A technique to solve inhomogeneous equations using Sturm-Liouville expansions, one dimensional heat and wave equations with inhomogeneous boundary conditions, one dimensional inhomogeneous heat and wave equations, mixed boundary conditions, Dirichlet problem in a rectangle and a polar coordinate rectangle

Maximum Principle and applications: maximum principle for linear, second-order, ordinary differential equations, generalized maximum principle for such equations, applications to initial and boundary value problems, the eigenvalue problem, an extension of the principle to non-linear equations

Orthogonal polynomials and their properties: Legendre polynomials, Legendre equation, Legendre functions and spherical harmonics, Hermite polynomials, Hermite functions, Hermite equation, Laguerre polynomials, Laguerre equation, zeros of orthogonal polynomials on an interval, and a recurrence relation satisfied by them

Bessel Functions: Bessel's equation, identities, asymptotics and zeros of Bessel functions

***Suggested Books*:**

Texts:

- Birkhoff, G & Rota G., *Ordinary Differential Equations*, John Wiley & Sons
- Folland, G., *Fourier Analysis & Its Applications,* AMS
- Protter, M. & Weinberger, H.*, Maximum Principles in Differential Equations*, Springer

References:

- Brown, J. & Churchill, R., *Fourier Series and Boundary Value Problems*, McGraw-Hill
- Jackson, D., *Fourier Series and Orthogonal Polynomials*, Dover

MTH 510: Operator Theory and Operator Algebras (4)

***Pre-requisites**: MTH 503 Functional Analysis*

Course Contents:

Banach Algebras, Ideals, Quotients, homomorphisms, Unitization

Invertible Elements, Spectrum, Gelfand-Mazur Theorem, Spectral Radius Formula

Commutative Banach Algebras, The Gelfand Transform, Applications to Fourier Transforms, Weiner's Theorem, Stone-Weierstrass Theorem

Compact and Fredholm Operators, Atkinson's Theorem, Index Theory

C* algebras, uniqueness of the norm, Commutative C* algebras, Gelfand-Naimark theorem, Spectral Mapping theorem

Functional Calculus, Positive Operators, Polar Decomposition

Weak and Strong Operator Topologies, Von Neumann Algebras, Double Commutant Theorem

Spectral measure, Spectral Theorem for Normal Operators, Borel Functional Calculus

Multiplicity Theory, Abelian Von Neumann Algebras, Classification of normal operators upto unitary equivalence

***Suggested Books*:**

- G. J. Murphy, *C* Algebras and Operator Theory* (Academic Press Inc, 1990)
- J. B. Conway, *A Course in Functional Analysis* (2nd Ed) (Springer, 1990)
- R. G. Douglas, *Banach Algebra Techniques in Operator Theory* (2nd Ed) (Springer, 1998)
- K. R. Davidson, *C* Algebras by Example* (Fields Institute Monograph, AMS 1996)
- R. V. Kadison and J. R. Ringrose, *Fundamentals of the Theory of Operator Algebras - Vol. I* (Academic Press Inc, 1983)
- W. A. Arveson, *A Short Course in Spectral Theory* (Springer 2002)

MTH 511: Introduction to Lie Groups and Lie Algebras (4)

Pre-requisites*: Required : MTH 311, Desirable : MTH 301* Groups and Rings

Learning Objectives**:**

The proposed course aims at providing and introduction to Lie groups, Lie algebras and their representations. The first part of the course focuses on matrix Lie groups (closed subgroups of GL(n; C)) and Lie algebras. The second part of the course deals with representations of semisimple Lie groups and Lie algebras. We begin with SU(2) and SU(3), as these cases very well illustrate the ideas of Cartansubalgebras, the roots, weights and the Wey1 group. We also look at Semisimple Lie groups and Lie algebras in general towards the end.

Course Contents**:**

Matrix Lie Groups: Definition and examples; Lie group homomorphisms and isomorphisms, Lie subgroups, polar decomposition.

Lie algebras: matrix exponential and matrix logarithm (4), one parameter subgroups, the Lie algebra of a matrix Lie group, Lie subalgebras, complexification of a real Lie algebra

Banker-Campbell-Hausborffformula : Definition, computations on Heisenberg group, Integral form of B-C-H formula (no proof), the series form of the B-C-H formula (no proof), applications to exponential map

Representations of SU(3): Weights and roots, theorem of the highest weight, the Wey1 group, weight diagrams

Semisimple Lie algebras: Complete reducibility, examples of reductive and semisimple Lie algebras, Cartansubalgebras, roots and root spaces, inner products of roots and co-roots, the Wey1 group, root systems, positive roots, the example of sl(n,C) in detail, uniqueness results.

Suggested Books**:**

- Hall, Brian Lie Groups, *Lie Algebras, and Representations. Graduate Texts in Mathematics*, Vol. 222, Springer Verlag, 2003.
- Rossmann, Wulf. *Lie Groups: An Introduction through Linear Groups*. Oxford Graduate Texts in Mathematics 5, Oxford University Press, 2002.
- Humphreys, James E. *Introduction to Lie Alogebras and Representation Theory. Graduate Texts in Mathematics,* Vol. 9, Springer, 1973.
- Baker, Andrew*. Matrix Groups: An Introduction to Lie Group Theory*. Springer Veriag, 2002.

MTH 512: Non-commutative Algebra (4)

***Pre-requisites**: MTH 301, MTH 302, MTH 401*

Matrix Rings and PLIDs, Tensor Products of Matrix Algebras, Ring constructions using Regular Representation

Basic notions for Noncommutative Rings, Structure of Hom(M,N), Semisimple Modules & Rings, the Wedderburn Structure Theorem, Simple Rings, Rings with Involution

The Jacobson Radical and its properties, Primitive Rings and Ideals, Hopkins-Levitzki Theorem, Nakayama's Lemma , Radical of a Module, Local Rings, Chevalley-Jacobson Theorem, Kolchin's Theorem, Clifford Algebras.

Prime and Semiprime rings, Rings of Fractions and Goldie's Theorems, Rings with ACC (ideals), Tensor Algebras, Algebras over large Fields, Deformations and Quantum Algebras.

Hereditary Rings and their Modules, Division rings.

Central Simple Algebras, Cyclic Algebras, Symbol Algebras, Crossed Products, the Brauer Group, the functor Br, the Skolem-Noether Theorem, the centralizer Theorem, calculation of Brauer group of commutative rings.

***Suggested Books*:**

- L. Rowen, Graduate algebra: noncommutative view, Graduate Studies in Mathematics, 91.
- B. Farb, R. Dennis, Noncommutative algebra, GTM, Springer-Verlag.
- T. Y. Lam, A first course in noncommutative rings, GTM, Springer.
- J. Golan and T. Head, Modules and the structure of rings: A primer, Pure and applied mathematics

MTH 513: Introduction to Riemannian Geometry (4)

Pre-requisites*: MTH 405 and MTH 508*

Review of differentiable manifolds: vector bundles, tensors, vector fields, differential forms, Lie groups

Riemannian metrics. Definition, examples, existence theorem; model spaces of Riemannian geometry

Connections: connections on a vector bundle, linear connections, covariant derivative, parallel transport, geodesics

Riemannian connections and geodesics: torsion tensor, Fundamental Theorem of Riemannian Geometry, geodesics of the model spaces, exponential map, convex neighborhoods, Riemannian distance function, first variation formula, Gauss' lemma, geodesics as locally minimizing curves; completeness, statement of Hopf-Rinow Theorem

Curvature: Riemann Curvature Tensor, Bianchi identity, scalar, sectional and Ricci curvatures

Jacobi Fields: Jacobi equation, conjugate points, second variation formula, spaces of constant curvature (if time permits)

Curvature and topology: Gauss-Bonnet Theorem, Bonnet-Myers Theorem, Cartan-Hadamard Theorem

Suggested Books**:**

Texts:

- J. M. Lee. Riemannian Manifolds, An introduction to Curvature. Graduate Texts in Mathematics. Springer (1997).
- M. P. do Carmo. Riemannian Geometry. Birkhauser (1991).
- S. Gallot, D. Hulin, J. Lafontaine. Riemannian Geometry. Springer (2004).

References:

- I. Chavel. Riemannian geometry, a modern introduction. Cambridge University Press (2006)
- S. Kobayashi, K. Nomizu. Foundations of differential geometry, vol. -I, Wiley Interscience Publication (1996).

MTH 514: Representation Theory (4)

Pre-requisites*: MTH 301, MTH 302*

Representations of groups, subrepresentations, Irreducible representations, tensor product of representations, Maschke's theorem, Wedderburn decomposition

Characters of representations, Generalized characters, Schurs lemma, Orthogonality, Regular representations, Decomposition theorems

Representations of direct product of finite groups, Induced representations, Reciprocity theorem

Representations and characters of standard finite and infinite groups: cyclic groups, dihedral groups, symmetric and alternating groups of small order etc.

Applications of Representation Theory

***Suggested Books*:**

- J. P. Serre, *Linear Representations of Finite groups* (Graduate Texts in Mathematics), 2nd Edition, Springer-Verlag New York Inc., 1977
- W. Fulton and J. Harris, *Representation Theory*, *A First Course*, 2nd Indian reprint, Springer India, 2007
- *G. James, M. Liebeck, Representations and Characters of Groups*, Cambridge University Press, 2001
- M. Suzuki, *Group Theory II*, Springer-Verlag, 1983

MTH 516: Topology II (4)

***Pre-requisites (Desirable)**: MTH 507 or MTH 605 and MTH 302 or MTH 601*

***Learning Objectives*:**

This is an advanced course in Topology.

***Course Contents*:**

Simplicial Homology: Simplicial Complexes, Barycentric Subdivision, and Simplicial Homology with examples

Singular and Cellular Homology: Definition with examples, Homotopy Invariance, Exact Sequence of Relative Homology, Excision, Mayer-Vietoris Sequence, Degree of Maps, and Cellular Homology, Jordan-Brouwer Separation Theorem, Invariance of domain and dimension, Borsuk-UlamTheorem, Lefschetz-Hopf Fixed Point Theorem, Axioms for homology, Fundamental group and homology, and Simplicial Approximation Theorem

Cohomology: Universal Coefficient Theorem, Künneth Formula, Cup Product and the Cohomology Ring, Cap Product, Orientations on Manifolds, and Poincaré Duality
Higher Homotopy Groups: Definition with examples, Aspherical Spaces, Relative Homotopy Groups, Long Exact Sequence of a triple, n-connected spaces, and Whitehead's Theorem

***Suggested Books*:**

- A. Hatcher, Algebraic Topology, Cambridge University Press, 2002.
- E. H. Spanier, Algebraic Topology, Springer, 1994.
- J. R. Munkres, Elements of Algebraic Topology, Westview Press, 1996.
- J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988.
- M. J. Greenberg & J. R. Harper, Algebraic Topology: A First Course, Perseus Books Publishing, 1981.
- W. S. Massey, A Basic Course in Algebraic Topology, Springer International Edition, 2007.
- G. Bredon, Topology and Geometry, Springer International Edition.

MTH 517: Introduction to Algebraic Geometry (4)

Pre-requisites (Desirable)*: MTH 301, MTH 302, and MTH 401*

Learning Objectives**:**

This course aims to provide an introduction to some of the basic objects and techniques and objects of algebraic geometry with minimal prerequisites. The main emphasis is on geometrical ideas and so most of the treatment will be over algebraically closed fields. Results from commutative algebra will introduced and proved as required and so no prior experience with commutative algebra will be assumed. After introducing the basic objects and techniques, they will be illustrated by application to the theory of algebraic curves.

Course Contents**:**

Closed subsets of affine space, coordinate rings, correspondence between ideals and closed subsets, affine varieties, regular maps, rational functions, Hilbert's nullstellensatz

Projective and quasi-projective varieties, regular and rational functions on projective varieties, products and maps of quasi-projective varieties

Dimension of varieties, examples and applications

Local ring of a point, tangent and cotangent space, local parameters, non-singular points and non-singular varieties

Birational maps, blowups, disingularization of curves

Intersection numbers for plane curves, divisors on curves, Bezout's theorem, Riemann-Roch theorem for curves, Residue theorem, Riemann-Hurwitz formula

Suggested Books**:**

- W. Fulton, Algebraic curves: An introduction to algebraic geometry, 2008 ed. (available online).
- R. Shafarevich, Basic Algebraic Geometry, Vol. 1, Third Edition, Springer, Heidelberg, 2013.
- S. Abhyankar, Algebraic geometry for scientists and engineers, Mathematical Surveys and Monographs 35, American Mathematical Society, 1990.
- K. Smith et al, An invitation to algebraic geometry, Springer, 2004.

MTH 518: Commutative Algebra (4)

Pre-requisites (Desirable)*: MTH 401: Fields and Galois Theory*

Learning Objectives**:**

The aim of this course is to introduce commutative algebra. This theory has developed not just as a standalone area of algebra, but also as a tool to study other important branches of Mathematics including Algebraic Geometry and Algebraic Number Theory.

Course Contents**:**

- Quotient Rings, Prime and Maximal ideals, units, Nilradical, Jacobson Radical, Operations on ideals, Extensions and contractions
- Tensor product of Algebras (only existence theorem), Rings and Modules of fractions, Local properties, Structure passing between R and $S^{-1}R$ (resp. M and $S^{-1}M$)
- Primary decompositions, Uniqueness theorems, Chain conditions, Noetherian and Artinian Rings, Lasker-Noether theorem, Hilbert basis theorem, Nakayama's lemma, Krull intersection theorem
- Integral dependence, Going up theorem, Integrally closed integral domains, Going down theorem
- Valuation rings, Discrete valuation rings, Dedekind domains, Fractional ideals
- Valuations, Completions, Extensions of absolute values, residue field, Local fields, Ostrowski's theorem
- Hilbert's Nullstellensatz

Suggested Books**:**

- Introduction to Commutative Algebra, Atiyah, M and Macdonald, I.G., Levant Books, Kolkata
- Graduate Algebra: Commutative View, Rowen, L.H., Graduate Studies in Mathematics, AMS
- Commutative Algebra with a view towards Algebraic Geometry, Eisenbud, D., Springer

MTH 519: Introduction to Modular Forms (4)

Pre-requisites (Desirable)*: MTH 407: Complex Analysis I*

Learning Objectives**:**

The aim of this course is to introduce the theory of modular forms with minimal prerequisites. The course is intended for the students who have done the standard courses in Linear Algebra and Complex Analysis. The results and techniques from these courses will be used to understand the space of modular forms and hence the students will solidify their understandings of some basic tools learned throughout mathematics. Numerous examples of modular forms will be given which are useful in solving some classical problems in number theory. The purpose is to make the modular form theory accessible without going into the advanced algebraically oriented treatments of the subject. At the same time this course introduces the topics which are at the forefront of the current research.

Course Contents**:**

- The full modular group $SL_2(Z)$, Congruence subgroups, The upper half-plane H, Action of groups on H, Fundamental domains, The invariant metric on H
- Modular forms of integral weight of level one, Eisenstein series, The Ramanujan τ-function, Dedekind η-function, Poincare series, The valence formula and dimension formula, Modular forms of integral weight of higher level
- The Petersson inner product, Hecke operators, Oldforms and newforms, Dirichlet series associated to modular forms: Convergence, Analytic continuation, Functional equation
- (if time permits) Modular forms of half-integral weight: Definition and examples, Hecke operators, Shimura-Shintani correspondences between modular forms of integral weight and half-integral weight.

Suggested Books**:**

- M. Ram Murty, M. Dewar, H. Graves, Problems in the theory of modular forms, Institute of Mathematical Sciences - Lecture Notes 1, Hindustan Book Agency, 2015.
- N. Koblitz, Introduction to elliptic curves and modular forms, Graduate Texts in Mathematics 97, Springer, 1993.
- J. P. Serre, A course in arithmetic, Graduate Texts in Mathematics 7, Springer, 1973.
- T. M. Apostol, Modular functions and Dirichlet series in number theory, GTM 41, Springer, 1990.
- H. Iwaniec, Topics in classical automorphic forms, Graduate Studies in Mathematics 17, AMS, 1997.
- F. Diamond and J. Shurman, A first course in modular forms, Graduate Texts in Mathematics 228, Springer, 2005.
- T. Miyake, Modular forms, Springer Monographs in Mathematics, Springer, 2006.
- G. Shimura, Modular forms: basics and beyond, Springer Monographs in Mathematics, Springer, 2012.

MTH 520: Introduction to Hyperbolic Geometry (4)

***Pre-requisites (Desirable)**: MTH 304, MTH 407*

***Learning Objectives*:**

Hyperbolic geometry is arguably the most important area in modern geometry and topology. This course is intended to expose the student to the foundational concepts in hyperbolic geometry, and is specially tailored to prepare the student for advance topics in geometric topology.

***Course Contents*:**

The general Möbius group. The extended complex plane (or the Riemman sphere) **C**; The general Möbius group **Mob(Ĉ)**; Identifying **Mob^+(Ĉ)** with the matrix group **PGL(2; C);** Classification of elements of elements of **Mob^+(Ĉ)**; Reflections and the general Möbius group **Mob(Ĉ)**; Conformality of elements in **Mob(Ĉ)**.

The upper-half plane model H^2. The upper half plane**H^2**; The subgroup **Mod(H2)**; Transitivity properties of **$Mob^+(H^2)$**; Geometry of the action of **$Mob^+(H^2)$**; The metric in**H^2**; Element of arc-length in**H^2**; Path metric in**H^2**; The Poincaré metric **d_H** on**H^2**; Geodesics in**H^2**; Identifying the group **$Mob^+(H^2)$** of isometries of (**H^2, d_H)** with **PSL(2; R)**; Ultraparallel lines in **H^2**.

The Poincaré disk model D. The Poincaré disk **D**; Transitioning from**H^2**to D via **$Mob^+(H^2)$**; Element of arc-length and the metric **d_D** in **D**; The Group **Mob(D)** of isometries of **(D, d_D)**; Centre, radii, and length of hyperbolic circles in **D**; Hyperbolic structures on holomorphic disks.

Properties of H^2. Curvature of **H^2**; Convex subsets of **H^2**; Hyperbolic polygons; Area of a subset of **H^2**; Gauss-Bonnet formula - area of a hyperbolic triangle; Applications of Gauss-Bonnet Formula: Area of reasonable hyperbolic polygons, existence of certain hyperbolic *n*-gons, hyperbolic dilations; Putting a hyperbolic structure on a surface using hyperbolic polygons; Hyperbolic trigonometry: triogometric identities, law of sines and cosines, Pythagorean theorem.

Non-planar models (if time permits). Hyperboloid model for the hyperbolic plane; Higher dimensional hyperbolic spaces.

***Suggested Books*:**

- James W. Anderson, *Hyperbolic Geometry* (2nd Edition), Springer, 2005.
- Arlan Ramsay, Robert D. Richtmyer, *Introduction to Hyperbolic Geometry*, Springer, 1995.
- Harold E. Wolfe, *Introduction to Non-Euclidean Geometry*, Dover, 2012
- Alan F. Beardon, *The geometry of discrete groups* (Chapter 7), Springer, 1983.
- Svetlana Katok, *Fuchsian Groups* (Chapter 1), Chicago Lectures in Mathematics, 1992.
- John Stillwell, *Geometry of surfaces* (Chapter 4), Springer, 1992.

MTH 521: Introduction to Wavelets (4)

Pre-requisites (Desirable)*: MTH 311, MTH 404*

Learning Objectives**:**

This is an introductory course on wavelet analysis. In this course we will introduce the basic notion of wavelets in different settings, namely for finite groups, discrete infinite groups and real line. This will provide the students an opportunity to know perspective applications of linear algebra and real analysis in mathematics and beyond.

Course Contents**:**

Review of Linear Algebra: Complex Series, Euler's Formula, Roots of Unity, Linear Transformations and Matrices, Change of Basis, diagonalization of Linear Transformations and Matrices, Inner Product, Orthogonal Bases, Unitary Matrices.

The Discrete Fourier Transform: Definition and Basic Properties of Discrete Fourier Transform, Translation-Invariant Linear Transformations, The Fast Fourier Transform.

Wavelets on Finite Group Z_N: Convolution on Z_N, Fourier Transform on Z_N, Definition of Wavelets and Basic Properties, Construction of Wavelets on Z_N.

Wavelets on Infinite Discrete Group Z : Definition and Basic Properties of Hilbert spaces, Complete orthonormal Sets in Hilbert Spaces, The spaces $l_2(Z)$ and $L^2([-\pi, \pi))$, Basic Fourier Series, The Fourier Transform and Convolution on $l_2(Z)$ Wavelets on Z.

Wavelets on R: Convolution and Approximate Identities, Fourier Transform on R, Bases for The Space $L^2(R)$, Belian-Low Theorem, Wavelets on R, Multiresolution Analysis, Construction of Wavelets from multiresolution Analysis, Construction of Compactly supported Wavelets, Haar Wavelets, Band-Limited Wavelets, Applications.

Suggested Books**:**

- Michael W. Frazier: An Introduction to Wavelets Through Linear Algebra, Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1999.
- Eugenio Hernandez, Guido Weiss: A First Course on Wavelets, Studies in Advanced Mathematics. CRC Press, Boca Raton, FL, 1996.
- Ingrid, Daubechies: Ten Lectures on Wavelets, CBMS-NSF Regional Conference Series in Applied Mathematics, 61. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1992

MTH 522: Numerical Analysis (4)

***Pre-requisites**: MTH 303 Real Analysis I*

Round off errors and computer arithmetic

Interpolation: Lagrange interpolation, divided differences, Hermite interpolation, splines, numerical differentiation, Richardson extrapolation

Numerical Integration: trapezoidal, Simpsons, Newton-Cotes, Gauss quadrature, Romberg integration, multiple integrals

Solution of linear algebraic equations: direct methods, Gauss elimination, pivoting, matrix factorizations

Iterative methods: matrix norms, Jacobi and Gauss-Siedel methods, relaxation methods

Computation of eigenvalues and eigenvectors: power method, householders method, QR algorithm

Numerical solutions of non-linear algebraic equations: bisection, secan and Newton's, zeroes of polynomials

***Suggested Books*:**

- R. L. Burden, D. J. Faires, *Numerical Analysis*
- E. K. Blum, *Numerical Analysis and Computation*, *Theory and Practice,* Dover, 2010
- S. D. Conte, C. De Boor, *Elementary Numerical Analysis*,third edition, McGraw-Hill, 1980
- D. M. Young, R. T. Gregory, *A Survey of Numerical Mathematics*, volumes 1 and 2, Dover, 1988

MTH 524: An introduction to schemes and cohomology (4)

Prerequisites: MTH 415
Desirable: MTH 517
Course contents:

Basics of Category theory , Sheaves , Schemes, Properties of Schemes, Separated and Proper Morphisms, Sheaves of Modules, Divisors, Projective Morphisms, Differentials, Derived Functors, Cohomology of Sheaves, Cohomology of a Noeitherian Affine Schemes, CechCohomology, Cohomology of Projective spaces, Ext group and Scheves , Serre Duality theorem (statement only), Flat Morphisms and Smooth Morphisms

***Suggested Books*:**

- David Mumford,The Red Book of Varieties and Schemes, Springer; 2nd exp. ed. 1999.
- Hartshorne, Robin. Algebraic Geometry. New York, NY: Springer, 1997.
- Phillip Griffiths, Joseph Harris, Principles of Algebraic Geometry, Wiley-Interscience; 1st edition 1994.

***References*:**

- David Eisenbud, The Geometry of Schemes: 197, Springer 2002
- Ulrich Görtz ,TorstenWedhorn, Geometry I: Schemes: With Examples and Exercises,Vieweg+Teubner Verlag,2010
- Shafarevich, Igor .R ,Basic Algebraic Geometry 2 : Schemes and Complex Manifolds, Springer Nature (SIE), 2014

MTH 601: Algebra I (4)

Monoids, Groups, group actions, Sylow's theorems, Finitely generated abelian groups, free groups

Rings and homomorphisms, Chinese remainder theorem, examples as polynomial ring and power series ring, rings of endomorphisms, Universal property of polynomial rings, Localization, Principal and factorial rings

Modules, quotient modules, direct product and direct sum of modules, Jordan-Hölder theorem, Free, Projective and Injective modules, Dual modules, Modules over PID

Category and Functor, Direct and Inverse limit

Polynomials in one and several variables, Gauss lemma, Irreducibility criterions, Power series ring, group of units in power series ring

Algebraic extensions, Algebraic closure, Splitting fields, Normal extensions, Separable and inseparable extensions, Finite fields

Galois extensions, examples, Galois correspondence theorems, solvability of equations

***Suggested Books*:**

- Algebra (3rd Edition), Serge Lang, Addison Welsey
- Basic Algebra, Jacobson, parts-I and II, Dover Publications Inc.; 2nd edition.
- Algebra, Birkhoff and McLane, Chelsea Publishing Co.
- A course in theory of groups, D.J.S. Robinson, Springer; 2nd ed.

MTH 602: Algebra II (4)

Extension of rings, integral extensions, going up and going down theorems, integral closure, integral galois extensions

Transcendental extension, transcendence basis, Noether normalization theorem, separable and regular extensions

Algebraic varieties, Hilbert's Nullstellensatz, Spec of a ring

Noetherian (and Artinian) rings and Modules

Matrices and linear maps, determinants, duality, bilinear and quadratic forms

Tensor product, basic properties, bimodules, Flat modules, extension of scalars, Algebras, Graded algebras, Tensor, symmetric and exterior algebras

***Suggested Books*:**

- S. Lang, Algebra, 3rd Edition, Addison Wesley.
- Jacobson, Basic Algebra I and II.
- Birkhoff and McLane, Algebra.
- Dummit and Foote, Abstract Algebra, 2nd Edition, Wiley.

MTH 603: Real Analysis (4)

Several variable calculus: A quick overview, the contraction mapping theorem, the inverse function theorem, the implicit function theorem.
Riemann integration in $\mathbf{R}^n$, $n \geq 1$.

Lebesgue measure and integration: Measures, measurable functions, integration of nonnegative and complex functions, modes of convergence, convergence theorems, product measure, Fubini's theorem, convolution, integration in polar coordinates.

Signed measures and differentiation, complex measures, total variation, absolute continuity, Fundamental theorem of calculus for Lebesgue integral, the Radon-Nikodym theorem and consequences.

$\mathbf{L}^p$ spaces, the Hölder and Minkowski inequalities, Jensen's inequality, completeness, the Riesz representation theorem, dual of $\mathbf{L}^p$ spaces.

***Suggested Books*:**

- G.B. Folland, Real analysis: Modern techniques and their applications, 2nd Edition, Wiley.
- W. Rudin, Principles of Mathematical Analysis, 3rd Edition, Tata McGraw-Hill.
- W. Rudin, Real and Complex Analysis, 3rd Edition, Tata McGraw-Hill.
- E.M. Stein and R. Shakarchi, Functional Analysis: Introduction to further topics in analysis, Princeton lectures in analysis.
- T. Tao, Analysis I and II, 2nd Edition, TRIM Series 37, 38, Hindustan Book Agency.

MTH 604: Complex Analysis II (4)

Pre-requisites*:*
Required: MTH 303 Real Analysis I, MTH 407 Complex Analysis
Desirable: MTH 304 Metric Spaces and Topology, MTH 503 Functional Analysis

Review of elementary concepts: Complex differentiation, Cauchy-Riemann equations, holomorphicity, complex integration, Cauchy's theorem and Cauchy's integral formula, Taylor and Laurent series, residue theorem, definition of a meromorphic function.

Harmonic functions: definition and properties, Poisson integral formula, mean-value property, Schwarz reflection principle, Dirichlet problem

Maximum modulus principle: Maximum modulus theorem, Schwarz lemma, Phragmen-Lindelof theorem

Approximations by rational functions: Runge's theorem, Mittag-Leffler theorem

Conformal mappings: definition and examples, space of holomorphic functions, Montel's theorem, statement of Riemann mapping theorem

Entire functions, Infinite products, Weierstrass factorization theorem, little and big Picard Theorems, Gamma function

Suggested Books**:**

Texts:

- Stein E.M. and Shakarchi R., *Complex Analysis (Princeton Lectures in Analysis Series, Vol. II)*, Princeton University Press, 2003
- Conway J.B., *Functions of One Complex Variable*, Springer-Verlag NY, 1978
- Rudin W., *Real and Complex Analysis*, McGraw-Hill, 2006
- Epstein B. and Hahn L-S., Classical Complex Analysis, Jones and Bartlett, 2011
- Ahlfors L., *Complex Analysis*, Lars Ahlfors, McGraw-Hill, 1979.

References:

- Carathodory C., *Theory of functions of a complex variable, Vol. I and II*, Chelsea Pub Co, NY 1954
- Remmert R., Classical topics in complex function theory, Springer 1997

MTH 605: Topology I (4)

General Topology: Connectedness, Compactness, Local Compactness, Paracompactness, Quotient Spaces, Topological Groups, and Baire Category Theorem.

Fundamental Gruops and Covering Spaces: Homotopy, Homotopy Equivalence and Deformation Retractions, Fundamental Group, Van Kampen Theorem, Deck Transformations, Group Actions, and Classification of covering spaces.

Cellular and Simplicial Complexes: Operations on Cell Complexes and Homotopy Extension Property. Simplicial Complexes - Barycentric Subdivision and Simplicial Approximation Theorem.

***Suggested Books*:**

- A. Hatcher, Algebraic Topology, Cambridge University Press, 2002.
- J. R. Munkres, Topology, Second Edition, Prentice Hall, 2011.
- G. Bredon, Topology and Geometry, Springer International Edition, 2006.
- W. S. Massey, A Basic Course in Algebraic Topology, Springer International Edition, 2007.
- J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988.
- J. R. Munkres, Elements of Algebraic Topology, Westview Press, 1996.

MTH 606: Ordinary Differential Equations (4)

***Pre-requisites**: MTH 201, MTH 303, MTH 403*

First-order equations

- Direction fields, approximate solutions , the fundamental inequality, uniqueness and existence theorems, solutions of equations containing parameters
- Comparison theorems

Systems of first-order equations

- Linear systems with constant coefficients: exponentials of linear operators, the fundamental theorem for linear systems, linear systems in the plane, canonical forms of linear operators on a complex vector space (S+N decomposition, nilpotent canonical forms, Jordan and real canonical forms), stability theory (saddle, spiral, and nodal points), phase portraits
- Linear equations of higher order: fundamental systems, Wronskian, reduction of order, non-homogeneous linear systems, Green's function
- Non-linear systems: the fundamental existence-uniqueness theorem, dependence on initial conditions and parameters, the maximal interval of existence, the flow defined by a differential equation, linearization, the Stable Manifold theorem, the Hartman-Grobman theorem, stability theory of equilibria (saddles, nodes, foci and centres), Liapunov functions, La-Salle's invariance principle, gradient systems

Poincare-Bendixson theory: Limit sets, local sections, theorem of Poincare-Bendixson, Poincare's index, orbital stability of limit cycles, index of simple singularities

***Suggested Books*:**

- G. Birkhoff& G. C. Rota, Ordinary differential equations, Paperback edition, John Wiley & Sons, 1989
- W. Hurewicz, Lectures on ordinary differential equations, Dover, New York, 1997
- Morris Hirsch and Stephen Smale, Differential Equations, Dynamical Systems, and Linear Algebra (Pure and Applied Mathematics (Academic Press), 1974
- P. Hartman, Ordinary Differential Equations, New York, Wiley, 1964

MTH 607: Complex Analysis I (4)

Pre-requisites (Desirable)*: MTH 303: Real Analysis I*

Learning Objectives**:**

The aim of this course is to introduce the theory of modular forms with minimal prerequisites. The learning objectives of this course include the definition of analyticity, the Cauchy-Riemann equations and the concept of differentiability. Also to be learnt are the theorems on entire functions, residue theorem and applications and finally conformal mapping.

Course Contents**:**

- Complex numbers: powers and roots, geometric representation, stereographic projection
- Complex differentiability: limits, continuity and differentiability, Cauchy Riemann equations, definition of a holomorphic function
- Elementary functions: sequences and series, complex exponential, trigonometric, and hyperbolic functions, the logarithm function, complex powers, Mobius transformations
- Complex integration: contour integrals, Cauchy's integral theorem in a disc, Cauchy’s Integral Formula, Liouville’s theorem, Fundamental Theorem of Algebra, Morera’s theorem, Schwarz reflection principle
- Series representation of analytic functions: Taylor series, power series, zeros and singularities, Laurent decomposition, open mapping theorem, Maximum Principle
- Residue theory: residue formula, calculation of certain improper integrals, Riemann’s theorem on removable singularities, CasoratiWeierstrass theorem, the argument principle and Rouche's theorem
- Conformal mappings: conformal maps, Schwarz lemma and automorphisms of the disk and the upper half plane

Suggested Books**:**

Texts

- Elias M. Stein, Rami Shakarchi, Complex Analysis, Princeton University Press, 2003
- Theodore W. Gamelin, Complex Analysis, Springer Verlag, 2001
- John B. Conway, Functions of one Complex Variable I, Springer, 1978
- E. Freitag and R.Busam, Complex Analysis, Springer, 2005

References

- Lars Ahlfors, Complex Analysis. McGrawHill, 1979
- R. Remmert, Theory of Complex Functions. Springer Verlag, 1991
- C. Caratheodory, Theory of Functions of a complex variable, AMS Chelsea, 2001

MTH 608: Introduction to Differentiable Manifolds and Lie Groups (4)

***Pre-requisites**: MTH 303 Real Analysis I, MTH 304 Metric Spaces and Topology, MTH 306 Ordinary Differential Equations, MTH 403 Real Analysis II*

Differentiable manifolds: definition and examples, differentiable functions, existence of partitions of unity, tangent vectors and tangent space at a point, tangent bundle, differential of a smooth map, inverse function theorem, implicit function theorem, immersions, submanifolds, submersions, Sard's theorem, Whitney embedding theorem

Vector fields: vector fields, statement of the existence theorem for ordinary differential equations, one parameter and local one-parameter groups acting on a manifold, the Lie derivative and the Lie algebra of vector fields, distributions and the Frobenius theorem

Lie groups: definition and examples, action of a Lie group on a manifold, definition of Lie algebra, the exponential map, Lie subgroups and closed subgroups, homogeneous manifolds: definition and examples

Tensor fields and differential forms: cotangent vectors and the cotangent space at a point, cotangent bundle, covector fields or 1-forms on a manifold, tensors on a vector space, tensor product, symmetric and alternating tensors, the exterior algebra, tensor fields and differential forms on a manifold, the exterior algebra on a manifold

Integration: orientation of a manifold, a quick review of Riemann integration in Euclidean spaces, differentiable simplex in a manifold, singular chains, integration of forms over singular chains in a manifold, manifolds with boundary, integration of n-forms over regular domains in an oriented manifold of dimension n, Stokes theorem, definition of de Rhamcohomology of a manifold, statement of de Rham theorem, Poincare lemma

***Suggested Books*:**

Texts:

- J. Lee, *Introduction to smooth manifolds*, Springer, 2002
- W. Boothby, *An Introduction to differentiable manifolds and Riemannian geometry,* Academic Press, 2002
- F. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer, GTM 94, 1983
- M. Spivak, *A comprehensive introduction to differential geometry, Vol. 1*, Publish or Perish, 1999

References:

- G. de Rham, *Differentiable manifolds: forms, currents and harmonic forms*, Springer, 1984
- V. Guillemin and A. Pollack., *Differential topology,* AMS Chelsea, 2010
- J. Milnor, *Topology from the differentiable viewpoint,* Princeton University Press, 1997
- J. Munkres, *Analysis on manifolds,* Westview Press, 199
- 01254217
- C. Chevalley, *Theory of Lie groups,* Princeton University Press, 1999
- R. Abraham, J. Marsden, T. Ratiu, *Manifolds, tensor analysis, and applications*, Springer, 1988

MTH 609: Sturm-Liouville Theory (4)

***Pre-requisites**: MTH 306 Ordinary Differential Equations, MTH 404 Measure and Integration*

Fourier Series: Fourier series of a periodic function, question of point-wise convergence of such a series, behavior of the Fourier series under the operation of differentiation and integration , sufficient conditions for uniform and absolute convergence of a Fourier series, Fourier series on intervals, examples of boundary value problems for the one dimensional heat and wave equations illustrating the use of Fourier series in solving them by separating variables, a brief discussion on Cesarosummability and Gibbs phenomenon

Orthogonal Expansions: A quick review of L2 spaces on an interval, convergence, completeness, orthonormal systems, Bessel's inequality, Parseval's identity, dominated convergence theorem
Sturm-Liouville Systems: linear differential operators, formal adjoint of a linear operator, Lagrange's identity, self-adjoint operators, regular and singular Sturm-Liouville systems, Sturm-Liouville series, Prufer substitution, Sturm comparison and oscillation theorems, eigenfunctions, Liouville normal form, distribution of eigenvalues, normalized eigenfunctions, Green's functions, completeness of eigenfunctions

Illustrative boundary value problems: A technique to solve inhomogeneous equations using Sturm-Liouville expansions, one dimensional heat and wave equations with inhomogeneous boundary conditions, one dimensional inhomogeneous heat and wave equations, mixed boundary conditions, Dirichlet problem in a rectangle and a polar coordinate rectangle

Maximum Principle and applications: maximum principle for linear, second-order, ordinary differential equations, generalized maximum principle for such equations, applications to initial and boundary value problems, the eigenvalue problem, an extension of the principle to non-linear equations

Orthogonal polynomials and their properties: Legendre polynomials, Legendre equation, Legendre functions and spherical harmonics, Hermite polynomials, Hermite functions, Hermite equation, Laguerre polynomials, Laguerre equation, zeros of orthogonal polynomials on an interval, and a recurrence relation satisfied by them

Bessel Functions: Bessel's equation, identities, asymptotics and zeros of Bessel functions

***Suggested Books*:**

Texts:

- Birkhoff, G & Rota G., *Ordinary Differential Equations*, John Wiley & Sons
- Folland, G., *Fourier Analysis & Its Applications,* AMS
- Protter, M. & Weinberger, H.*, Maximum Principles in Differential Equations*, Springer

References:

- Brown, J. & Churchill, R., *Fourier Series and Boundary Value Problems*, McGraw-Hill
- Jackson, D., *Fourier Series and Orthogonal Polynomials*, Dover

MTH 610: Fourier Analysis on the Real Line (4)

Pre-requisites*: MTH 404 Measure and Integration, MTH 503 Functional Analysis: Normed linear spaces, completeness, Uniform boundedness principle, MTH 405 Partial Differential Equations: Basic knowledge of Laplacian, Heat and Wave equations*

The vibrating string, derivation and solution to the wave equation, The heat equation

Definition of Fourier series and Fourier coefficients, Uniqueness, Convolutions, good kernels, Cesaro/Abel means, Poisson Kernel and Dirichlet's problem in the unit disc

Mean-square convergence of Fourier Series, Riemann-Lebesgue Lemma, A continuous function with diverging Fourier Series

Applications of Fourier Series : The isoperimetric inequality, Weyl's equidistribution Theorem, A continuous nowhere-differentiable function, The heat equation on the circle

Schwartz space*, Distributions*, The Fourier transform on **R**: Elementary theory and definition, Fourier inversion, Plancherel formula, Poisson summation formula, Paley-Weiner Theorem*, Heisenberg Uncertainty principle, Heat kernels, Poisson Kernels

(If time permits/possible project topic) Definition of Fourier transform on $\mathbf{R}^d$, Definition of X-ray transform in $\mathbf{R}^2$ and Radon transform in $\mathbf{R}^3$, Connection with Fourier Transform, Uniqueness

Suggested Books**:**

Texts:

- E.M.Stein and R. Shakarchi, *Fourier Analysis: An Introduction, Princeton Univ Press*, 2003
- (For topics marked with a*) W. Rudin, *Functional Analysis,* 2nd Ed, Tata McGraw-Hill, 2006

References:

- J. Douandikoetxea*, Fourier Analysis* (Graduate Studies in Mathematics), AMS, 2000
- L. Grafakos, *Classical Fourier Analysis* (Graduate Texts in Mathematics), 2nd Ed, Springer, 2008

MTH 612: Non-commutative Algebra (4)

***Pre-requisites**: MTH 301, MTH 302, MTH 401*

Matrix Rings and PLIDs, Tensor Products of Matrix Algebras, Ring constructions using Regular Representation

Basic notions for Noncommutative Rings, Structure of Hom(M,N), Semisimple Modules & Rings, the Wedderburn Structure Theorem, Simple Rings, Rings with Involution

The Jacobson Radical and its properties, Primitive Rings and Ideals, Hopkins-Levitzki Theorem, Nakayama's Lemma , Radical of a Module, Local Rings, Chevalley-Jacobson Theorem, Kolchin's Theorem, Clifford Algebras.

Prime and Semiprime rings, Rings of Fractions and Goldie's Theorems, Rings with ACC (ideals), Tensor Algebras, Algebras over large Fields, Deformations and Quantum Algebras.

Hereditary Rings and their Modules, Division rings.

Central Simple Algebras, Cyclic Algebras, Symbol Algebras, Crossed Products, the Brauer Group, the functor Br, the Skolem-Noether Theorem, the centralizer Theorem, calculation of Brauer group of commutative rings.

***Suggested Books*:**

- L. Rowen, Graduate algebra: noncommutative view, Graduate Studies in Mathematics, 91.
- B. Farb, R. Dennis, Noncommutative algebra, GTM, Springer-Verlag.
- T. Y. Lam, A first course in noncommutative rings, GTM, Springer.
- J. Golan and T. Head, Modules and the structure of rings: A primer, Pure and applied mathematics

MTH 613: Introduction to Riemannian Geometry (4)

***Pre-requisites**: MTH 405 and MTH 508*

Review of differentiable manifolds: vector bundles, tensors, vector fields, differential forms, Lie groups

Riemannian metrics. Definition, examples, existence theorem; model spaces of Riemannian geometry

Connections: connections on a vector bundle, linear connections, covariant derivative, parallel transport, geodesics

Riemannian connections and geodesics: torsion tensor, Fundamental Theorem of Riemannian Geometry, geodesics of the model spaces, exponential map, convex neighborhoods, Riemannian distance function, first variation formula, Gauss' lemma, geodesics as locally minimizing curves; completeness, statement of Hopf-Rinow Theorem

Curvature: Riemann Curvature Tensor, Bianchi identity, scalar, sectional and Ricci curvatures

Jacobi Fields: Jacobi equation, conjugate points, second variation formula, spaces of constant curvature (if time permits)

Curvature and topology: Gauss-Bonnet Theorem, Bonnet-Myers Theorem, Cartan-Hadamard Theorem

***Suggested Books*:**

Texts:

- J. M. Lee. Riemannian Manifolds, An introduction to Curvature. Graduate Texts in Mathematics. Springer (1997).
- M. P. do Carmo. Riemannian Geometry. Birkhauser (1991).
- S. Gallot, D. Hulin, J. Lafontaine. Riemannian Geometry. Springer (2004).

References:

- I. Chavel. Riemannian geometry, a modern introduction. Cambridge University Press (2006)
- S. Kobayashi, K. Nomizu. Foundations of differential geometry, vol. -I, Wiley Interscience Publication (1996).

MTH 614: Functional Analysis (4)

Pre-requisites (Desirable)*: MTH 404: Measure and Integration*

Learning Objectives**:**

Functional analysis is the branch of mathematics concerned with the study of spaces of functions. This course is intended to introduce the student to the basic concepts and theorems of functional analysis with special emphasis on Hilbert and Banach Space Theory. This gives the basics for more advanced studies in modern Functional Analysis, in particular in Operator Algebra Theory and Banach Space Theory.

Course Contents**:**

- Normed Linear spaces, Bounded Linear Operators, Banach Spaces, Finite dimensional spaces, Quotient Spaces
- Hilbert spaces, Riesz Representation Theorem, Orthonormal sets, Bessel's Inequality, Parseval's Identity, Fourier Series
- Dual Spaces, Dual of L^p spaces , Hahn-Banach Extension Theorem, Applications
- Open Mapping Theorem, Closed Graph Theorem, Uniform Boundedness Principle
- Weak and Weak-* topologies, Hahn-Banach Separation Theorem, Alaoglu's Theorem, Reflexivity
- Compact Operators, Adjoint of an operator, Spectral theorem for Compact Self-Adjoint operators
- (If time permits) Banach Algebras, Ideals and Quotients, Gelfand-Mazur Theorem, Fredholm Alternative, Fredholm Operators, Atkinson's theorem

Suggested Books**:**

- J.B. Conway, A Course in Functional Analysis, 2nd Ed., (Springer-Verlag, 1990)
- S. Kesavan, Functional Analysis, TRIM 52, Hindustan Book Agency
- B.V. Limaye, Functional Analysis, 2nd Ed., (New Age International, 1996)
- Martin Schechter, Principles of Functional Analysis, 2nd Ed., Graduate Studies in Mathematics, AMS
- P.D Lax, Functional Analysis, (Wiley, 2002)
- W. Rudin, Functional Analysis, 2nd Ed., (Tata McGraw-Hill, 2006)

MTH 615: Operator Theory and Operator Algebras (4)

***Pre-requisites**: MTH 503 Functional Analysis*

Course Contents:

Banach Algebras, Ideals, Quotients, homomorphisms, Unitization

Invertible Elements, Spectrum, Gelfand-Mazur Theorem, Spectral Radius Formula

Commutative Banach Algebras, The Gelfand Transform, Applications to Fourier Transforms, Weiner's Theorem, Stone-Weierstrass Theorem

Compact and Fredholm Operators, Atkinson's Theorem, Index Theory

C* algebras, uniqueness of the norm, Commutative C* algebras, Gelfand-Naimark theorem, Spectral Mapping theorem

Functional Calculus, Positive Operators, Polar Decomposition

Weak and Strong Operator Topologies, Von Neumann Algebras, Double Commutant Theorem

Spectral measure, Spectral Theorem for Normal Operators, Borel Functional Calculus

Multiplicity Theory, Abelian Von Neumann Algebras, Classification of normal operators upto unitary equivalence

***Suggested Books*:**

- G. J. Murphy, *C* Algebras and Operator Theory* (Academic Press Inc, 1990)
- J. B. Conway, *A Course in Functional Analysis* (2nd Ed) (Springer, 1990)
- R. G. Douglas, *Banach Algebra Techniques in Operator Theory* (2nd Ed) (Springer, 1998)
- K. R. Davidson, *C* Algebras by Example* (Fields Institute Monograph, AMS 1996)
- R. V. Kadison and J. R. Ringrose, *Fundamentals of the Theory of Operator Algebras - Vol. I* (Academic Press Inc, 1983)
- W. A. Arveson, *A Short Course in Spectral Theory* (Springer 2002)

MTH 616: Topology II (4)

Pre-requisites (Desirable)*: MTH 507 or MTH 605 and MTH 302 or MTH 601*

Learning Objectives**:**

This is an advanced course in Topology.

Course Contents**:**

Simplicial Homology: Simplicial Complexes, Barycentric Subdivision, and Simplicial Homology with examples

Singular and Cellular Homology: Definition with examples, Homotopy Invariance, Exact Sequence of Relative Homology, Excision, Mayer-Vietoris Sequence, Degree of Maps, and Cellular Homology, Jordan-Brouwer Separation Theorem, Invariance of domain and dimension, Borsuk-UlamTheorem, Lefschetz-Hopf Fixed Point Theorem, Axioms for homology, Fundamental group and homology, and Simplicial Approximation Theorem

Cohomology: Universal Coefficient Theorem, Künneth Formula, Cup Product and the Cohomology Ring, Cap Product, Orientations on Manifolds, and Poincaré Duality
Higher Homotopy Groups: Definition with examples, Aspherical Spaces, Relative Homotopy Groups, Long Exact Sequence of a triple, n-connected spaces, and Whitehead's Theorem

Suggested Books**:**

- A. Hatcher, Algebraic Topology, Cambridge University Press, 2002.
- E. H. Spanier, Algebraic Topology, Springer, 1994.
- J. R. Munkres, Elements of Algebraic Topology, Westview Press, 1996.
- J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988.
- M. J. Greenberg & J. R. Harper, Algebraic Topology: A First Course, Perseus Books Publishing, 1981.
- W. S. Massey, A Basic Course in Algebraic Topology, Springer International Edition, 2007.
- G. Bredon, Topology and Geometry, Springer International Edition.

MTH 617: Introduction to Algebraic Geometry (4)

***Learning Objectives*:**

This course aims to provide an introduction to some of the basic objects and techniques and objects of algebraic geometry with minimal prerequisites. The main emphasis is on geometrical ideas and so most of the treatment will be over algebraically closed fields. Results from commutative algebra will introduced and proved as required and so no prior experience with commutative algebra will be assumed. After introducing the basic objects and techniques, they will be illustrated by application to the theory of algebraic curves.

***Course Contents*:**

Closed subsets of affine space, coordinate rings, correspondence between ideals and closed subsets, affine varieties, regular maps, rational functions, Hilbert's nullstellensatz

Projective and quasi-projective varieties, regular and rational functions on projective varieties, products and maps of quasi-projective varieties

Dimension of varieties, examples and applications

Local ring of a point, tangent and cotangent space, local parameters, non-singular points and non-singular varieties

Birational maps, blowups, disingularization of curves

Intersection numbers for plane curves, divisors on curves, Bezout's theorem, Riemann-Roch theorem for curves, Residue theorem, Riemann-Hurwitz formula

***Suggested Books*:**

- W. Fulton, Algebraic curves: An introduction to algebraic geometry, 2008 ed. (available online).
- R. Shafarevich, Basic Algebraic Geometry, Vol. 1, Third Edition, Springer, Heidelberg, 2013.
- S. Abhyankar, Algebraic geometry for scientists and engineers, Mathematical Surveys and Monographs 35, American Mathematical Society, 1990.
- K. Smith et al, An invitation to algebraic geometry, Springer, 2004.

MTH 618: Commutative Algebra (4)

Pre-requisites (Desirable): *MTH 401: Fields and Galois Theory*

***Learning Objectives*:**

The aim of this course is to introduce commutative algebra. This theory has developed not just as a standalone area of algebra, but also as a tool to study other important branches of Mathematics including Algebraic Geometry and Algebraic Number Theory.

***Course Contents*:**

- Quotient Rings, Prime and Maximal ideals, units, Nilradical, Jacobson Radical, Operations on ideals, Extensions and contractions
- Tensor product of Algebras (only existence theorem), Rings and Modules of fractions, Local properties, Structure passing between R and $S^{-1}R$ (resp. M and $S^{-1}M$)
- Primary decompositions, Uniqueness theorems, Chain conditions, Noetherian and Artinian Rings, Lasker-Noether theorem, Hilbert basis theorem, Nakayama's lemma, Krull intersection theorem
- Integral dependence, Going up theorem, Integrally closed integral domains, Going down theorem
- Valuation rings, Discrete valuation rings, Dedekind domains, Fractional ideals
- Valuations, Completions, Extensions of absolute values, residue field, Local fields, Ostrowski's theorem
- Hilbert's Nullstellensatz

***Suggested Books*:**

- Introduction to Commutative Algebra, Atiyah, M and Macdonald, I.G., Levant Books, Kolkata
- Graduate Algebra: Commutative View, Rowen, L.H., Graduate Studies in Mathematics, AMS
- Commutative Algebra with a view towards Algebraic Geometry, Eisenbud, D., Springer

MTH 619: Introduction to Modular Forms (4)

Pre-requisites (Desirable)*: MTH 407: Complex Analysis I*

Learning Objectives**:**

The aim of this course is to introduce the theory of modular forms with minimal prerequisites. The course is intended for the students who have done the standard courses in Linear Algebra and Complex Analysis. The results and techniques from these courses will be used to understand the space of modular forms and hence the students will solidify their understandings of some basic tools learned throughout mathematics. Numerous examples of modular forms will be given which are useful in solving some classical problems in number theory. The purpose is to make the modular form theory accessible without going into the advanced algebraically oriented treatments of the subject. At the same time this course introduces the topics which are at the forefront of the current research.

Course Contents**:**

- The full modular group $SL_2(Z)$, Congruence subgroups, The upper half-plane H, Action of groups on H, Fundamental domains, The invariant metric on H
- Modular forms of integral weight of level one, Eisenstein series, The Ramanujan τ-function, Dedekind η-function, Poincare series, The valence formula and dimension formula, Modular forms of integral weight of higher level
- The Petersson inner product, Hecke operators, Oldforms and newforms, Dirichlet series associated to modular forms: Convergence, Analytic continuation, Functional equation
- (if time permits) Modular forms of half-integral weight: Definition and examples, Hecke operators, Shimura-Shintani correspondences between modular forms of integral weight and half-integral weight.

Suggested Books**:**

- M. Ram Murty, M. Dewar, H. Graves, Problems in the theory of modular forms, Institute of Mathematical Sciences - Lecture Notes 1, Hindustan Book Agency, 2015.
- N. Koblitz, Introduction to elliptic curves and modular forms, Graduate Texts in Mathematics 97, Springer, 1993.
- J. P. Serre, A course in arithmetic, Graduate Texts in Mathematics 7, Springer, 1973.
- T. M. Apostol, Modular functions and Dirichlet series in number theory, GTM 41, Springer, 1990.
- H. Iwaniec, Topics in classical automorphic forms, Graduate Studies in Mathematics 17, AMS, 1997.
- F. Diamond and J. Shurman, A first course in modular forms, Graduate Texts in Mathematics 228, Springer, 2005.
- T. Miyake, Modular forms, Springer Monographs in Mathematics, Springer, 2006.
- G. Shimura, Modular forms: basics and beyond, Springer Monographs in Mathematics, Springer, 2012.

MTH 620: Introduction to Hyperbolic Geometry (4)

Pre-requisites (Desirable)*: MTH 304, MTH 407*

Learning Objectives**:**

Hyperbolic geometry is arguably the most important area in modern geometry and topology. This course is intended to expose the student to the foundational concepts in hyperbolic geometry, and is specially tailored to prepare the student for advance topics in geometric topology.

Course Contents**:**

The general Möbius group. The extended complex plane (or the Riemman sphere) **C**; The general Möbius group **Mob(Ĉ)**; Identifying $\mathbf{Mob^+(\hat{C})}$ with the matrix group **PGL(2; C);** Classification of elements of elements of $\mathbf{Mob^+(\hat{C})}$; Reflections and the general Möbius group **Mob(Ĉ)**; Conformality of elements in **Mob(Ĉ)**.

The upper-half plane model $\mathbf{H^2}$. The upper half plane $\mathbf{H^2}$; The subgroup **Mod(H2)**; Transitivity properties of $\mathbf{Mob^+(H^2)}$; Geometry of the action of $\mathbf{Mob^+(H^2)}$; The metric in $\mathbf{H^2}$; Element of arc-length in $\mathbf{H^2}$; Path metric in $\mathbf{H^2}$; The Poincaré metric $\mathbf{d_H}$ on $\mathbf{H^2}$; Geodesics in $\mathbf{H^2}$; Identifying the group $\mathbf{Mob^+(H^2)}$ of isometries of $(\mathbf{H^2, d_H})$ with **PSL(2; R)**; Ultraparallel lines in $\mathbf{H^2}$.

The Poincaré disk model D. The Poincaré disk **D**; Transitioning from $\mathbf{H^2}$ to D via $\mathbf{Mob^+(H^2)}$; Element of arc-length and the metric $\mathbf{d_D}$ in **D**; The Group **Mob(D)** of isometries of $\mathbf{(D,\ d_D)}$; Centre, radii, and length of hyperbolic circles in **D**; Hyperbolic structures on holomorphic disks.

Properties of $\mathbf{H^2}$. Curvature of $\mathbf{H^2}$; Convex subsets of $\mathbf{H^2}$; Hyperbolic polygons; Area of a subset of $\mathbf{H^2}$; Gauss-Bonnet formula - area of a hyperbolic triangle; Applications of Gauss-Bonnet Formula: Area of reasonable hyperbolic polygons, existence of certain hyperbolic *n*-gons, hyperbolic dilations; Putting a hyperbolic structure on a surface using hyperbolic polygons; Hyperbolic trigonometry: triogometric identities, law of sines and cosines, Pythagorean theorem.
Non-planar models (if time permits). Hyperboloid model for the hyperbolic plane; Higher dimensional hyperbolic spaces.

Suggested Books**:**

- James W. Anderson, *Hyperbolic Geometry* (2nd Edition), Springer, 2005.
- Arlan Ramsay, Robert D. Richtmyer, *Introduction to Hyperbolic Geometry*, Springer, 1995.
- Harold E. Wolfe, *Introduction to Non-Euclidean Geometry*, Dover, 2012
- Alan F. Beardon, *The geometry of discrete groups* (Chapter 7), Springer, 1983.
- Svetlana Katok, *Fuchsian Groups* (Chapter 1), Chicago Lectures in Mathematics, 1992.
- John Stillwell, *Geometry of surfaces* (Chapter 4), Springer, 1992.

MTH 621: Introduction to Wavelets (4)

Pre-requisites (Desirable)*: MTH 311, MTH 404*

Learning Objectives**:**

This is an introductory course on wavelet analysis. In this course we will introduce the basic notion of wavelets in different settings, namely for finite groups, discrete infinite groups and real line. This will provide the students an opportunity to know perspective applications of linear algebra and real analysis in mathematics and beyond.

Course Contents**:**

Review of Linear Algebra: Complex Series, Euler's Formula, Roots of Unity, Linear Transformations and Matrices, Change of Basis, diagonalization of Linear Transformations and Matrices, Inner Product, Orthogonal Bases, Unitary Matrices.

The Discrete Fourier Transform: Definition and Basic Properties of Discrete Fourier Transform, Translation-Invariant Linear Transformations, The Fast Fourier Transform.

Wavelets on Finite Group Z_N: Convolution on Z_N, Fourier Transform on Z_N, Definition of Wavelets and Basic Properties, Construction of Wavelets on Z_N.

Wavelets on Infinite Discrete Group Z : Definition and Basic Properties of Hilbert spaces, Complete orthonormal Sets in Hilbert Spaces, The spaces $l_2(Z)$ and $L^2([-\pi, \pi))$, Basic Fourier Series, The Fourier Transform and Convolution on $l_2(Z)$ Wavelets on Z.

Wavelets on R: Convolution and Approximate Identities, Fourier Transform on R, Bases for The Space $L^2(R)$, Belian-Low Theorem, Wavelets on R, Multiresolution Analysis, Construction of Wavelets from multiresolution Analysis, Construction of Compactly supported Wavelets, Haar Wavelets, Band-Limited Wavelets, Applications.

Suggested Books**:**

- Michael W. Frazier: An Introduction to Wavelets Through Linear Algebra, Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1999.
- Eugenio Hernandez, Guido Weiss: A First Course on Wavelets, Studies in Advanced Mathematics. CRC Press, Boca Raton, FL, 1996.
- Ingrid, Daubechies: Ten Lectures on Wavelets, CBMS-NSF Regional Conference Series in Applied Mathematics, 61. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1992

MTH 623: Introduction to Ergodic Theory (4)

Pre-requisites*: MTH 304 Metric Spaces and Topology, MTH 404 Measure and Integration*

Discrete Dynamical systems: definition and examples - maps on the circle, the doubling map, shifts of finite type, toralautomorphisms.

Topological and Symbolic dynamics: transitivity, minimality, topological conjugacy and discrete spectrum, topological mixing, topological entropy, topological dynamical properties of shift spaces, circle maps and rotation number.

Ergodic Theory: invariant measures and measure-preserving transformations, ergodicity, recurrence and ergodic theorems (Poincare recurrence, Kac's lemma, Von Neumann's ergodic theorem, Birkhoff's ergodic theorem), applications of the ergodic theorem (continued fractions, Borel normal numbers, Khintchine's recurrence theorem), ergodic measures for continuous transformations and their existence, Weyl's equidistribution theorem, mixing and spectral properties.

Information and entropy - topological, measure-theoretic, and their relationship. Skew products, factors and natural extensions, induced transformations, suspensions and towers. Topological pressure and the variational principle, thermodynamic formalism and transfer operators, applications of thermodynamic formalism: (i) Bowen's formula for Hausdorff dimension, (ii) central limit theorems.

***Suggested Books*:**

- P. Walters, An Introduction to Ergodic Theory, Springer-Verlag, New York, 1982
- M.G. Nadkarni, Basic Ergodic Theory, Second Edition, Hindustan Book Agency, India
- M. Brin and G. Stuck, Introduction to Dynamical Systems, CUP, 2002
- M. Pollicott and M. Yuri, Dynamical systems and Ergodic theory, CUP, 1998
- P. R. Halmos, Lectures on Ergodic Theory, Chelsea, New York, 1956
- W. Parry, B. Bollobas, W. Fulton, Topics in Ergodic Theory, CUP, 2004
- A.B. Katok and B. Hasselblatt, Introduction to the Modern Theory of Dynamical Systems, Cambridge, 1995

MTH 624: An introduction to schemes and cohomology (4)

Prerequisites: MTH 415
Desirable: MTH 517
Course contents:

Basics of Category theory , Sheaves , Schemes, Properties of Schemes, Separated and Proper Morphisms, Sheaves of Modules, Divisors, Projective Morphisms, Differentials, Derived Functors, Cohomology of Sheaves, Cohomology of a Noeitherian Affine Schemes, CechCohomology, Cohomology of Projective spaces, Ext group and Scheves , Serre Duality theorem (statement only), Flat Morphisms and Smooth Morphisms

***Suggested Books*:**

- David Mumford,The Red Book of Varieties and Schemes, Springer; 2nd exp. ed. 1999.
- Hartshorne, Robin. Algebraic Geometry. New York, NY: Springer, 1997.
- Phillip Griffiths, Joseph Harris, Principles of Algebraic Geometry, Wiley-Interscience; 1st edition 1994.

***References*:**

- David Eisenbud, The Geometry of Schemes: 197, Springer 2002
- Ulrich Görtz ,TorstenWedhorn, Geometry I: Schemes: With Examples and Exercises,Vieweg+Teubner Verlag,2010
- Shafarevich, Igor .R ,Basic Algebraic Geometry 2 : Schemes and Complex Manifolds, Springer Nature (SIE), 2014